Cuaderno de notas de trabajo

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Cuaderno 51

Principios Variacionales.

Variación de la ds de la Relatividad Especial.

Hipótesis:

1) las variaciones libres son las de las coordenadas.

2) la variación de ds es función de las variaciones de las coordenadas.

$$ds^2 = \Delta_{ij}\, dx^i dx^j$$

$$2\, ds\, \delta(ds) = 2\, \Delta_{ij}\, dx^i\, d(\delta x^j)$$

$$\delta(ds) = \Delta_{ij} \frac{dx^i}{ds}\, d(\delta x^j)$$

$$\frac{dx^i}{ds} = v^i$$

$$\delta(ds) = \Delta_{ij}\, v^i\, d(\delta x^j)$$

$$\delta(ds) = \hat{v} \cdot d(\delta \hat{r})$$

¡ $\delta \hat{r}$ es una variación libre !

Variación del cuadrivector velocidad.

$$\delta v^i = \delta \frac{dx^i}{ds} = \frac{d(\delta x^i)}{ds} - dx^i \frac{\hat{v} \cdot d(\delta \hat{r})}{ds^2}$$

$$\delta v^i = \frac{d}{ds}(\delta x^i) - v^i \left(\hat{v} \cdot \frac{d}{ds} \delta \hat{r} \right)$$

$$\boxed{\delta \hat{v} = \frac{d}{ds}(\delta \hat{r}) - \left(\hat{v} \cdot \frac{d}{ds}(\delta \hat{r}) \right) \hat{v}}$$

$$\boxed{\delta v^i = \frac{d}{ds}(\delta x^i) - \left[\Delta_{jk} v^j \frac{d}{ds}(\delta x^k) \right] v^i}$$

$$L\,ds = A + A_{i_1} dx^{i_1} + A_{i_1 i_2} v^{i_1} dx^{i_2} + A_{i_1 i_2 i_3} v^{i_1} v^{i_2} dx^{i_3} + A_{i_1 i_2 i_3 i_4} v^{i_1} v^{i_2} v^{i_3} dx^{i_4} + \dots$$

$$L\,ds = \sum_i A_{i_1 i_2 i_3 \dots i_{n-1} i_n} v^{i_1} v^{i_2} v^{i_3} \dots v^{i_{n-1}} dx^{i_n}$$

$$L\,ds = \sum_i A_{i_1 i_2 i_3 \dots i_{n-1} i_n} v^{i_1} v^{i_2} v^{i_3} \dots v^{i_{n-1}} v^{i_n} ds$$

$$A_{i_1 i_2 i_3 \dots i_\alpha \dots i_\beta \dots i_{n-1} i_n} = A_{i_1 i_2 i_3 \dots i_\beta \dots i_\alpha \dots i_{n-1} i_n}$$

$$\delta A_{i_1 i_2 i_3} v^{i_1} v^{i_2} v^{i_3} ds = \left\{ \frac{\partial A_{i_1 i_2 i_3}}{\partial x^j} v^{i_1} v^{i_2} v^{i_3} \delta x^j ds + 3 A_{i_1 i_2 i_3} v^{i_1} v^{i_2} d(\delta x^{i_3}) \right.$$
$$\left. - 3 A_{i_1 i_2 i_3} \Delta_{i_4 i_5} v^{i_1} v^{i_2} v^{i_3} v^{i_4} d(\delta x^{i_5}) \right.$$
$$\left. + A_{i_1 i_2 i_3} v^{i_1} v^{i_2} v^{i_3} \Delta_{i_4 i_5} v^{i_4} d(\delta x^{i_5}) \right)$$

$$\delta A_{i_1 i_2 i_3} v^{i_1} v^{i_2} v^{i_3} ds = \Big[\frac{\partial A_{i_1 i_2 i_3}}{\partial x^{i_4}} v^{i_1} v^{i_2} v^{i_3} \delta x^{i_4} ds + 3 A_{i_1 i_2 i_3} v^{i_1} v^{i_2} d(\delta x^{i_3}) - 2 A_{i_1 i_2 i_3} \Delta_{i_4 i_5} v^{i_1} v^{i_2} v^{i_3} v^{i_4} d(\delta x^{i_5}) \Big]$$

$$\delta A_{i_1 i_2 i_3 \dots i_n} v^{i_1} v^{i_2} v^{i_3} \dots v^{i_n} ds = \Big[\frac{\partial A_{i_1 i_2 \dots i_n}}{\partial x^{i_{n+1}}} v^{i_1} v^{i_2} \dots v^{i_n} \delta x^{i_{n+1}} ds + n A_{i_1 i_2 \dots i_n} v^{i_1} v^{i_2} \dots v^{i_{n-1}} d(\delta x^{i_n}) - (n-1) A_{i_1 i_2 \dots i_n} \Delta_{i_{n+1} i_{n+2}} v^{i_1} v^{i_2} \dots v^{i_n} v^{i_{n+1}} d\delta x^{i_{n+2}} \Big]$$

$(d \delta x^{i_{n+2}})$

Variación del elemento de arco ds.

$$ds^2 = \Delta_{ij}\, dx^i dx^j$$

$$2\,ds\,\delta(ds) = 2\,\Delta_{ij}\, dx^i\, d(\delta x^j)$$

$$\delta(ds) = \Delta_{ij}\, \frac{dx^i}{ds}\, d(\delta x^j)$$

$$\boxed{\delta(ds) = \Delta_{ij}\, v^i\, d(\delta x^j)}$$

$$\boxed{\delta(ds) = \hat{v} \cdot d(\delta \hat{r})}$$

Variación del cuadrivector velocidad $\hat{v}$

$$v^i = \frac{dx^i}{ds}$$

$$\delta v^i = \frac{d}{ds}(\delta x^i) - \frac{dx^i}{ds^2}\left[\hat{v} \cdot d(\delta \hat{r})\right]$$

$$\boxed{\delta v^i = \frac{d}{ds}(\delta x^i) - v^i\left[\Delta_{jk}\, v^j \frac{d}{ds}(\delta x^k)\right]}$$

$$\boxed{\delta \hat{v} = \frac{d}{ds}(\delta \hat{r}) - \hat{v}\left[\hat{v} \cdot \frac{d}{ds}(\delta \hat{r})\right]}$$

$$A_{i_1 i_2 i_3} v^{i_1} v^{i_2} v^{i_3}$$

$$A_{111} v^1 v^1 v^1 + A_{112} v^1 v^1 v^2 + A_{113} v^1 v^1 v^3 + A_{114} v^1 v^1 v^4$$
$$+ A_{121} v^1 v^2 v^1 + A_{122} v^1 v^2 v^2 + A_{123} v^1 v^2 v^3 + A_{124} v^1 v^2 v^4$$
$$+ A_{131} v^1 v^3 v^1 + A_{132} v^1 v^3 v^2 + A_{133} v^1 v^3 v^3 + A_{134} v^1 v^3 v^4$$
$$+ A_{141} v^1 v^4 v^1 + A_{142} v^1 v^4 v^2 + A_{143} v^1 v^4 v^3 + A_{144} v^1 v^4 v^4$$
$$+ A_{211} v^2 v^1 v^1 + A_{212} v^2 v^1 v^2 + A_{213} v^2 v^1 v^3 + A_{214} v^2 v^1 v^4$$
$$+ A_{221} v^2 v^2 v^1 + A_{222} v^2 v^2 v^2 + A_{223} v^2 v^2 v^3 + A_{224} v^2 v^2 v^4$$
$$+ A_{231} v^2 v^3 v^1 + A_{232} v^2 v^3 v^2 + A_{233} v^2 v^3 v^3 + A_{234} v^2 v^3 v^4$$
$$+ A_{241} v^2 v^4 v^1 + A_{242} v^2 v^4 v^2 + A_{243} v^2 v^4 v^3 + A_{244} v^2 v^4 v^4$$
$$+ A_{311} v^3 v^1 v^1 + A_{312} v^3 v^1 v^2 + A_{313} v^3 v^1 v^3 + A_{314} v^3 v^1 v^4$$
$$+ A_{321} v^3 v^2 v^1 + A_{322} v^3 v^2 v^2 + A_{323} v^3 v^2 v^3 + A_{324} v^3 v^2 v^4$$
$$+ A_{331} v^3 v^3 v^1 + A_{332} v^3 v^3 v^2 + A_{333} v^3 v^3 v^3 + A_{334} v^3 v^3 v^4$$
$$+ A_{341} v^3 v^4 v^1 + A_{342} v^3 v^4 v^2 + A_{343} v^3 v^4 v^3 + A_{344} v^3 v^4 v^4$$
$$+ A_{411} v^4 v^1 v^1 + A_{412} v^4 v^1 v^2 + A_{413} v^4 v^1 v^3 + A_{414} v^4 v^1 v^4$$
$$+ A_{421} v^4 v^2 v^1 + A_{422} v^4 v^2 v^2 + A_{423} v^4 v^2 v^3 + A_{424} v^4 v^2 v^4$$
$$+ A_{431} v^4 v^3 v^1 + A_{432} v^4 v^3 v^2 + A_{433} v^4 v^3 v^3 + A_{434} v^4 v^3 v^4$$
$$+ A_{441} v^4 v^4 v^1 + A_{442} v^4 v^4 v^2 + A_{443} v^4 v^4 v^3 + A_{444} v^4 v^4 v^4$$

$$A_{i_1 i_2 i_3} v^{i_1} v^{i_2} v^{i_3}$$

$$A_{111}(v^1)^3 + A_{222}(v^2)^3 + A_{333}(v^3)^3 + A_{444}(v^4)^3$$
$$+3A_{112}(v^1)^2 v^2 + 3A_{113}(v^1)^2 v^3 + 3A_{114}(v^1)^2 v^4$$
$$+3A_{122} v^1 (v^2)^2 + 3A_{223}(v^2)^2 v^3 + 3A_{224}(v^2)^2 v^4$$
$$+3A_{133} v^1 (v^3)^2 + 3A_{233} v^2 (v^3)^2 + 3A_{334}(v^3)^2 v^4$$
$$+3A_{144} v^1 (v^4)^2 + 3A_{244} v^2 (v^4)^2 + 3A_{344} v^3 (v^4)^2$$
$$+6A_{123} v^1 v^2 v^3 + 6A_{124} v^1 v^2 v^4 + 6A_{134} v^1 v^3 v^4$$
$$+6A_{234} v^2 v^3 v^4$$

$$\frac{\partial}{\partial v^3} A_{i_1 i_2 i_3} v^{i_1} v^{i_2} v^{i_3} = \Big[3A_{333}(v^3)^2 + 3A_{113}(v^1)^2$$
$$+3A_{223}(v^2)^2 + 6A_{133} v^1 v^3$$
$$+6A_{233} v^2 v^3 + 6A_{334} v^3 v^4$$
$$+3A_{344}(v^4)^2 + 6A_{123} v^1 v^2$$
$$+6A_{134} v^1 v^4 + 6A_{234} v^2 v^4 \Big]$$

$$\frac{\partial}{\partial v^3} A_{i_1 i_2 i_3} v^{i_1} v^{i_2} v^{i_3} = 3 A_{i_1 i_2 3} v^{i_1} v^{i_2}$$

$$A_{i_1 i_2 3} v^{i_1} v^{i_2} = \Big[A_{113}(v^1)^2 + A_{123} v^1 v^2 + A_{133} v^1 v^3 + A_{134} v^1 v^4$$
$$+ A_{213} v^1 v^2 + A_{223}(v^2)^2 + A_{233} v^2 v^3 + A_{234} v^2 v^4$$
$$+ A_{313} v^1 v^3 + A_{323} v^2 v^3 + A_{333}(v^3)^2 + A_{334} v^3 v^4$$
$$+ A_{413} v^1 v^4 + A_{423} v^2 v^4 + A_{433} v^3 v^4 + A_{344}(v^4)^2$$

$$A_{i_1 i_2 3} v^{i_1} v^{i_2} = \Big[A_{113}(v^1)^2 + 2A_{123} v^1 v^2 + 2A_{133} v^1 v^3$$
$$+ 2A_{134} v^1 v^4 + A_{223}(v^2)^2$$
$$+ 2A_{233} v^2 v^3 + 2A_{234} v^2 v^4$$
$$+ A_{333}(v^3)^2 + 2A_{334} v^3 v^4 + A_{344}(v^4)^2$$

$$3 A_{i_1 i_2 3} v^{i_1} v^{i_2} = \Big[+ 3A_{333}(v^3)^2$$
$$3A_{113}(v^1)^2 + 3A_{223}(v^2)^2 + 3A_{344}(v^4)^2$$
$$+ 6A_{133} v^1 v^3 + 6A_{233} v^2 v^3$$
$$+ 6A_{334} v^3 v^4$$
$$+ 6A_{123} v^1 v^2 + 6A_{134} v^1 v^4$$
$$+ 6A_{234} v^2 v^4 \Big]$$

$$\frac{\partial}{\partial v^{i_3}} A_{i_1 i_2 i_3} v^{i_1} v^{i_2} v^{i_3} = 3 A_{i_1 i_2 i_3} v^{i_1} v^{i_2}$$

$$\frac{\partial}{\partial v^{i_1}} A_{i_1 i_2 i_3} v^{i_1} v^{i_2} v^{i_3} = 3 A_{i_1 i_2 i_3} v^{i_2} v^{i_3}$$

$$\frac{\partial}{\partial v^{i_1}} A_{i_1 i_2 i_3 \cdots i_{n-1} i_n} v^{i_1} v^{i_2} v^{i_3} \cdots v^{i_{n-1}} v^{i_n} =$$

$$n A_{i_1 i_2 i_3 \cdots i_{n-1} i_n} v^{i_2} v^{i_3} v^{i_4} \cdots v^{i_{n-1}} v^{i_n}$$

$$\delta\left[A_{i_1 i_2 i_3 \cdots i_{n-1} i_n} v^{i_1} v^{i_2} v^{i_3} \cdots v^{i_{n-1}} v^{i_n} ds\right] =$$

$$\frac{\partial A_{i_1 i_2 i_3 \cdots i_{n-1} i_n}}{\partial x^j} v^{i_1} v^{i_2} v^{i_3} \cdots v^{i_{n-1}} v^{i_n} \delta x^j ds$$

$$+ n A_{i_1 i_2 i_3 \cdots i_{n-1} i_n} v^{i_2} v^{i_3} \cdots v^{i_{n-1}} v^{i_n} d(\delta x^{i_1})$$

$$- n A_{i_1 i_2 i_3 \cdots i_{n-1} i_n} v^{i_1} v^{i_2} v^{i_3} \cdots v^{i_{n-1}} v^{i_n} \Delta_{i_{n+1} j} v^{i_{n+1}} d(\delta x^j)$$

$$+ A_{i_1 i_2 i_3 \cdots i_{n-1} i_n} v^{i_1} v^{i_2} v^{i_3} \cdots v^{i_{n-1}} v^{i_n} \Delta_{i_{n+1} j} v^{i_{n+1}} d(\delta x^j)$$

$$\delta\left[A_{i_1 i_2 i_3 \cdots i_{n-1} i_n} v^{i_1} v^{i_2} v^{i_3} \cdots v^{i_{n-1}} v^{i_n} ds\right] =$$

$$\frac{\partial A_{i_1 i_2 i_3 \cdots i_{n-1} i_n}}{\partial x^j} v^{i_1} v^{i_2} v^{i_3} \cdots v^{i_{n-1}} v^{i_n} \delta x^j ds$$

$$+ n A_{i_1 i_2 i_3 \cdots i_{n-1} j} v^{i_1} v^{i_2} v^{i_3} \cdots v^{i_{n-1}} d(\delta x^j)$$

$$- (n-1) A_{i_1 i_2 i_3 \cdots i_{n-1} i_n} \Delta_{i_{n+1} j} v^{i_1} v^{i_2} v^{i_3} \cdots v^{i_{n-1}} v^{i_n} v^{i_{n+1}} d(\delta x^j)$$

$$\delta \int A_{i_1 i_2 i_3 \ldots i_{n-1} i_n} v^{i_1} v^{i_2} v^{i_3} \ldots v^{i_{n-1}} v^{i_n} \, ds =$$

$$\int \Big[\frac{\partial A_{i_1 i_2 i_3 \ldots i_{n-1} i_n}}{\partial x^j} v^{i_1} v^{i_2} v^{i_3} \ldots v^{i_{n-1}} v^{i_n}$$

$$- n \frac{d}{ds} \Big\langle A_{i_1 i_2 i_3 \ldots i_{n-1} j} v^{i_1} v^{i_2} v^{i_3} \ldots v^{i_{n-1}} \Big\rangle$$

$$+ (n-1) \frac{d}{ds} \Big\langle A_{i_1 i_2 i_3 \ldots i_{n-1} i_n} \Delta_{i_{n+1} j} v^{i_1} v^{i_2} v^{i_3} \ldots v^{i_{n-1}} v^{i_n} v^{i_{n+1}} \Big\rangle \Big] (\delta x^j) \, ds$$

$$\frac{\partial A_{i_1 i_2 i_3 \cdots i_{n-1} i_n}}{\partial x^j} v^{i_1} v^{i_2} v^{i_3} \cdots v^{i_{n-1}} v^{i_n} \qquad \text{grado } n$$

$$- n \frac{\partial A_{i_1 i_2 i_3 \cdots i_{n-1} j}}{\partial x^{i_n}} v^{i_1} v^{i_2} v^{i_3} \cdots v^{i_{n-1}} v^{i_n} \qquad \text{grado } n$$

$$- n(n-1) A_{i_1 i_2 i_3 \cdots i_{n-1} j} a^{i_1} v^{i_2} v^{i_3} \cdots v^{i_{n-1}} \quad (\dagger) \qquad \text{grado } n$$

$$+ (n-1) \frac{\partial A_{i_1 i_2 i_3 \cdots i_{n-1} i_n}}{\partial x^{i_{n+1}}} \Delta_{i_{n+2} j} v^{i_1} v^{i_2} v^{i_3} \cdots v^{i_{n-1}} v^{i_n} v^{i_{n+1}} v^{i_{n+2}} \quad (*) \qquad \text{grado } n+2$$

$$+ n(n-1) A_{i_1 i_2 i_3 \cdots i_{n-1} i_n} \Delta_{i_{n+1} j} a^{i_1} v^{i_2} v^{i_3} \cdots v^{i_{n-1}} v^{i_n} v^{i_{n+1}} \quad (\star) \qquad \text{grado } n+2$$

$$+ (n-1) A_{i_1 i_2 i_3 \cdots i_{n-1} i_n} \Delta_{i_{n+1} j} v^{i_1} v^{i_2} v^{i_3} \cdots v^{i_{n-1}} v^{i_n} a^{i_{n+1}} \qquad \text{grado } n+2$$

(†)

$$A_{i_1 i_2 i_3 \dots i_{n-1} j}\, a^{i_1} v^{i_2} v^{i_3} v^{i_4} \dots v^{i_{n-1}}$$

$$A_{i_1 i_2 i_3 \dots i_{n-1} j}\, \Delta^{i_1 k} \left(\frac{\partial h_{kl}}{\partial x^m} - \frac{\partial h_{lm}}{\partial x^k} \right) v^l v^m v^{i_2} v^{i_3} v^{i_4} \dots v^{i_{n-1}}$$

$$A_{i_1 i_2 i_3 \dots i_{n-1} j}\, \Delta^{i_{n-1} k} \left(\frac{\partial h_k}{\partial x^k} - \frac{\partial h}{\partial x^k} \right) v^{i_1} v^{i_2} v^{i_3} \dots v^{i_n}$$

$$A_{i_1 i_2 i_3 \dots i_{n-2} j}$$

$$A_{i_1 i_2 i_3 \dots i_{n-2} p j}\, \Delta^{pk} \left(\frac{\partial h_{k\, i_{n-1}}}{\partial x^{i_n}} - \frac{\partial h_{i_{n-1} i_n}}{\partial x^k} \right) v^{i_1} v^{i_2} v^{i_3} \dots v^{i_{n-2}} v^{i_{n-1}} v^{i_n}$$

$$n(n-1)\, A_{i_1 i_2 i_3 \dots i_{n-1} j}\, a^{i_1} v^{i_2} v^{i_3} \dots v^{i_{n-1}} =$$

$$n(n-1)\, A_{i_1 i_2 i_3 \dots i_{n-2} p j}\, \Delta^{pk} \left(\frac{\partial h_{k\, i_{n-1}}}{\partial x^{i_n}} - \frac{\partial h_{i_{n-1} i_n}}{\partial x^k} \right) v^{i_1} v^{i_2} v^{i_3} \dots v^{i_{n-1}} v^{i_n}$$

(*)

$$(n-3)\ \frac{\partial A_{i_1 i_2 i_3 \ldots i_{n-2}}}{\partial x^{i_{n-1}}}\ \Delta^{\cdot}_{i_n j}\cdot v^{i_1} v^{i_2} v^{i_3} \ldots v^{i_{n-1}} v^{i_n}$$

$$A_{i_1 i_2 i_3 \cdots i_{n-1} i_n} \Delta_{i_{n+1} j} \Delta^{i_1 p} \left(\frac{\partial h_{pq}}{\partial x^r} - \frac{\partial h_{qr}}{\partial x^p} \right) v^q v^r v^{i_2} v^{i_3} v^{i_4} \cdots v^{i_{n-1}} v^{i_n} v^{i_{n+1}}$$

$$A_{i_1 i_2 i_3 \cdots i_{n-1} i_n} \Delta^{i_1 p} \left(\frac{\partial h_{pq}}{\partial x^r} - \frac{\partial h_{qr}}{\partial x^p} \right) v^q v^r v^{i_2} v^{i_3} v^{i_4} \cdots v^{i_{n-1}} v^{i_n} v_j$$

$$A_{i_1 i_2 i_3 \cdots i_{n-1} q} \Delta^{qp} \left(\frac{\partial h_{p i_n}}{\partial x^{i_{n+1}}} - \frac{\partial h_{i_n i_{n+1}}}{\partial x^p} \right) v^{i_1} v^{i_2} v^{i_3} \cdots v^{i_{n-1}} v^{i_n} v^{i_{n+1}} v_j$$

$$A_{i_1 i_2 i_3 \cdots i_{n-1} q} \Delta^{pq} \left(\frac{\partial h_{p i_n}}{\partial x^{i_{n+1}}} - \frac{\partial h_{i_n i_{n+1}}}{\partial x^p} \right) \Delta'_{j i_{n+2}} v^{i_1} v^{i_2} v^{i_3} \cdots v^{i_{n-1}} v^{i_n} v^{i_{n+1}} v^{i_{n+2}}$$

$$\boxed{(n-2)(n-3) A_{i_1 i_2 i_3 \cdots i_{n-3} q} \Delta^{pq} \Delta_{j i_n} \left(\frac{\partial h_{p i_{n-2}}}{\partial x^{i_{n-1}}} - \frac{\partial h_{i_{n-2} i_{n-1}}}{\partial x^p} \right) v^{i_1} v^{i_2} \cdots v^{i_{n-1}} v^{i_n}}$$

$$A_{i_1 i_2 i_3 \cdots i_{n-1} i_n} v^{i_1} v^{i_2} v^{i_3} \cdots v^{i_n} a_j$$

$$A_{i_1 i_2 i_3 \cdots i_{n-1} i_n} v^{i_1} v^{i_2} v^{i_3} \cdots v^{i_n} \left(\frac{\partial h_{j i_{n+1}}}{\partial x^{i_{n+2}}} - \frac{\partial h_{i_{n+1} i_{n+2}}}{\partial x^j} \right) v^{i_{n+1}} v^{i_{n+2}}$$

$$A_{i_1 i_2 i_3 \cdots i_{n-1} i_n} \left(\frac{\partial h_{j i_{n+1}}}{\partial x^{i_{n+2}}} - \frac{\partial h_{i_{n+1} i_{n+2}}}{\partial x^j} \right) v^{i_1} v^{i_2} v^{i_3} \cdots v^{i_n} v^{i_{n+1}} v^{i_{n+2}}$$

$$(n-3)\, A_{i_1 i_2 i_3 \cdots i_{n-2}} \left(\frac{\partial h_{j i_{n-1}}}{\partial x^{i_n}} - \frac{\partial h_{i_{n-1} i_n}}{\partial x^j} \right) v^{i_1} v^{i_2} v^{i_3} \cdots v^{i_n}$$

$$\left.\begin{aligned}
&\frac{\partial A_{i_1 i_2 i_3 \cdots i_{n-1} i_n}}{\partial x^j} - n \frac{\partial A_{i_1 i_2 i_3 \cdots i_{n-1} j}}{\partial x^{i_n}} \\
&- n(n-1) A_{i_1 i_2 i_3 \cdots i_{n-2} pj} \Delta^{pk} \left(\frac{\partial h_{k i_{n-1}}}{\partial x^{i_n}} - \frac{\partial h_{i_{n-1} i_n}}{\partial x^k} \right) \\
&+ (n-3) \frac{\partial A_{i_1 i_2 i_3 \cdots i_{n-2}}}{\partial x^{i_{n-1}}} \Delta_{i_n j} \\
&+ (n-2)(n-3) A_{i_1 i_2 i_3 \cdots i_{n-3} q} \Delta^{pq} \Delta_{j i_n} \left(\frac{\partial h_{p i_{n-2}}}{\partial x^{i_{n-1}}} - \frac{\partial h_{i_{n-2} i_{n-1}}}{\partial x^p} \right) \\
&+ (n-3) A_{i_1 i_2 i_3 \cdots i_{n-2}} \left(\frac{\partial h_{j i_{n-1}}}{\partial x^{i_n}} - \frac{\partial h_{i_{n-1} i_n}}{\partial x^j} \right)
\end{aligned}\right\} = 0$$

$$\mathcal{L}\,ds = A_{\cdot}\,ds + A_{ij}\,v^i dx^j + A_{ijkl}\,v^i v^j v^k dx^l + \dots$$

$$\underline{\delta\{\mathcal{L}\,ds\}}$$

$$\frac{\partial A_{\cdot}}{\partial x^p}\,\delta x^p ds + A_{\cdot}\,\Delta_{ip}\,v^i\,d(\delta x^p)$$

$$+ \frac{\partial A_{ij}}{\partial x^p}\,v^i v^j\,\delta x^p ds \,.$$

$$\mathcal{L}\,ds = \{ A\,ds + A_i\,dx^i + A_{ij}\,v^i\,dx^j + A_{ijk}\,v^i v^j\,dx^k + A_{ijkl}\,v^i v^j v^k\,dx^l + \dots \}$$

$$\delta \int \mathcal{L}\,ds$$

$$\frac{\partial A}{\partial x^p}\,\delta x^p\,ds + \cancel{\frac{\partial A_i}{\partial x^p}\,v^i\,\delta x^p\,ds} + A\,\Delta_{ip}\,v^i\,d(\delta x^p)$$

$$+ \frac{\partial A_i}{\partial x^p}\,v^i\,\delta x^p\,ds + A_p\,d(\delta x^p)$$

$$+ \frac{\partial A_{ij}}{\partial x^p}\,v^i v^j\,\delta x^p\,ds + A_{pj}\,v^j\,d(\delta x^p) + A_{ip}\,v^i\,d(\delta x^p)$$

$$+ \frac{\partial A_{ijk}}{\partial x^p}\,v^i v^j v^k\,\delta x^p\,ds + 2\,A_{ijk}\,v^i$$

$$\mathcal{L}_0 ds = A ds$$

$$\delta A ds.$$

$$\frac{\partial A}{\partial x^p} \delta x^p ds + A \Delta_{ip} v^i d(\delta x^p)$$

$$\frac{\partial A}{\partial x^p} - \frac{\partial A}{\partial x^j} \Delta_{ip} v^i v^j - A a_p$$

$$\frac{\partial A}{\partial x^p} - \frac{\partial A}{\partial x^j} \Delta_{ip} v^i v^j - A \left(\frac{\partial h_{pi}}{\partial x^j} - \frac{\partial h_{ij}}{\partial x^p} \right) v^i v^j$$

$$\frac{\partial A}{\partial x^p} - \left[\frac{\partial A}{\partial x^j} \Delta_{ip} - A \frac{\partial h_{pi}}{\partial x^j} + A \frac{\partial h_{ij}}{\partial x^p} \right] v^i v^j$$

$$\mathcal{L}_1 ds = A_i dx^i$$

$$\delta \mathcal{L}_1 ds = \delta A_i dx^i$$

$$\mathcal{L}_1 ds = A_i v^i ds \qquad \boxed{\mathcal{L}_1 = A_i v^i}$$

$$\mathcal{L}_1 ds = A_i dx^i$$

$$\delta A_i dx^i$$

$$\frac{\partial A_i}{\partial x^p} v^i \delta x^p ds + A_p d(\delta x^p)$$

$$\boxed{\frac{\partial A_i}{\partial x^p} v^i \delta x^p ds + A_p d(\delta x^p)}$$

$$\boxed{\delta A_i dx^i}$$

$$\boxed{\frac{\partial A_i}{\partial x^p} v^i \delta x^p ds + A_p d(\delta x^p)}$$

$$\frac{\partial A_i}{\partial x^p} v^i - \frac{\partial A_p}{\partial x^i} v^i$$

$$\boxed{\frac{\partial A_i}{\partial x^p} - \frac{\partial A_p}{\partial x^i}} v^i$$

$$\mathcal{L}_2 ds = A_{ij} v^i dx^j = A_{ij} v^i v^j ds$$

$$\mathcal{L}_2 = A_{ij} v^i v^j$$

$$\delta A_{ij} v^i dx^j$$

$$\frac{\partial A_{ij}}{\partial x^p} v^i v^j \delta x^p ds +$$

$$+ A_{pj} v^j d(\delta x^p) - A_{ij} v^i v^j \Delta_{pq} v^q d(\delta x^p)$$

$$+ A_{ip} v^i d(\delta x^p)$$

$$\delta A_{ij} v^i dx^j$$

$$\frac{\partial A_{ij}}{\partial x^p} v^i v^j \delta x^p ds +$$

$$+ 2A_{ip} v^i d(\delta x^p) - A_{ij} \Delta_{pq} v^i v^j v^q d(\delta x^p)$$

$$\frac{\partial A_{ij}}{\partial x^p} v^i v^j - 2\frac{\partial A_{ip}}{\partial x^j} v^i v^j - 2A_{ip} a^i$$

$$+ \frac{\partial A_{ij}}{\partial x^l} \Delta_{pk} v^i v^j v^k v^l + 2A_{ij} \Delta_{pk} a^i v^j v^k$$

$$+ A_{ij} \Delta_{pk} v^i v^j a^k$$

$$\left[\frac{\partial A_{ij}}{\partial x^p} - 2\frac{\partial A_{ip}}{\partial x^j} - 2A_{pq}\Delta^{qm}\left(\frac{\partial h_{mi}}{\partial x^j} - \frac{\partial h_{ij}}{\partial x^m}\right)\right] v^i v^j$$

$$+ \left[\frac{\partial A_{ij}}{\partial x^l}\Delta_{pk} + 2A_{jq}\Delta_{pk}\Delta^{qm}\left(\frac{\partial h_{mi}}{\partial x^l} - \frac{\partial h_{il}}{\partial x^m}\right)\right. \quad v^i v^j v^k v^l$$

$$\left. + A_{ij}\left(\frac{\partial h_{pk}}{\partial x^l} - \frac{\partial h_{kl}}{\partial x^p}\right)\right]$$

Introducción de una métrica completamente euclidiana.

(cartesiana).

~~$ds^2 = dx^2 + dy^2 - dz^2$~~

$$ds^2 = dt^2 - dx^2 - dy^2 - dz^2$$

$$-d\sigma^2 = -d\tau^2 - dx^2 - dy^2 - dz^2$$

$$\boxed{\tau = i\,t} \qquad \boxed{\sigma = i\,s} \qquad \boxed{ds^2 = -d\sigma^2}$$

$$d\sigma^2 = d\tau^2 + dx^2 + dy^2 + dz^2$$

$\tau \equiv x^1;\ x \equiv x^2;\ y \equiv x^3;\ z \equiv x^4$

$$\boxed{d\sigma^2 = \delta_{ij}\, dx^i\, dx^j}$$

$t \equiv X^1;\ x \equiv X^2;\ y \equiv X^3;\ z \equiv X^4$

$$\boxed{i\,X^1 = x^1}$$

$$\frac{d^2 X^1}{ds^2} = \Delta^{1p}\left[\frac{\partial h_{pj}}{\partial X^k} - \frac{\partial h_{jk}}{\partial X^p}\right]\frac{dX^j}{ds}\,\frac{dX^k}{ds}$$

$$-i\,\frac{d^2 X^1}{d\sigma^2} = i\left[\frac{\partial h_{1j}}{\partial X^k} - \frac{\partial h_{jk}}{\partial X^1}\right]\frac{dX^j}{ds}\,\frac{dX^k}{ds}$$

$$\frac{d^2x^1}{d\sigma^2} = i\,\frac{\partial h_{1j}}{\partial X^k}\frac{dX^j}{d\sigma}\frac{dX^k}{d\sigma} + \frac{\partial h_{jk}}{\partial x^1}\frac{dX^j}{d\sigma}\frac{dX^k}{d\sigma}$$

$$\frac{d^2x^\alpha}{ds^2} = \Delta^{\alpha p}\left[\frac{\partial h_{pj}}{\partial X^k} - \frac{\partial h_{jk}}{\partial X^p}\right]\frac{dX^j}{ds}\frac{dX^k}{ds}$$

$$\frac{d^2x^\alpha}{d\sigma^2} = -\frac{\partial h_{\alpha j}}{\partial x^k}\frac{dX^j}{d\sigma}\frac{dx^k}{d\sigma} + \frac{\partial h_{jk}}{\partial x^\alpha}\frac{dX^j}{d\sigma}\frac{dX^k}{d\sigma}$$

$$\begin{aligned} x^1 &= i\,X^1 \\ x^2 &= X^2 \\ x^3 &= X^3 \\ x^4 &= X^4 \end{aligned}$$

$$x = \begin{pmatrix} i & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} X$$

$$M = \begin{pmatrix} i & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \qquad M^{-1} = \begin{pmatrix} -i & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$h_{ij} = M^p_i\, M^q_j\, H_{pq}$$

$$h_{ij} = M^{-1}{}^p_i\, M^{-1}{}^q_j\, H_{pq}$$

$$v^i = \Lambda^i_j V^j \qquad v^i = \frac{\partial x^i}{\partial X^j} V^j$$

$$v_i = \frac{\partial X^j}{\partial x^i} V_j$$

$$h_{ij} = \frac{\partial X^p}{\partial x^i} \frac{\partial X^q}{\partial x^j} H_{pq}$$

$$h_{11} = - H_{11}$$

$$h_{1\alpha} = \frac{\partial X^1}{\partial x^1} \frac{\partial X^\alpha}{\partial x^\alpha} H_{1\alpha}$$

$$h_{1\alpha} = \frac{1}{i} H_{1\alpha} = - i H_{1\alpha}$$

$$\boxed{h_{1\alpha} = - i H_{1\alpha}} \qquad \boxed{h_{11} = - H_{11}}$$

$$\boxed{h_{\alpha\beta} = H_{\alpha\beta}}$$

$$\frac{d^2 X^1}{ds^2} = \left(\frac{\partial h_{1j}}{\partial X^k} \right) \frac{dX^j}{ds} \frac{dX^k}{ds} - \frac{\partial h_{jk}}{\partial X^1} \frac{dX^j}{ds} \frac{dX^k}{ds}$$

$$\frac{d^2 X^1}{d\sigma^2} = \left\{ \begin{aligned} & \frac{\partial h_{11}}{\partial X^1} \frac{dX^1}{d\sigma} \frac{dX^1}{d\sigma} + \frac{\partial h_{11}}{\partial X^\alpha} \frac{dX^1}{d\sigma} \frac{dX^\alpha}{d\sigma} \\ & + \frac{\partial h_{1\alpha}}{\partial X^1} \frac{dX^\alpha}{d\sigma} \frac{dX^1}{d\sigma} + \frac{\partial h_{1\alpha}}{\partial X^\beta} \frac{dX^\alpha}{d\sigma} \frac{dX^\beta}{d\sigma} \\ & - \end{aligned} \right.$$

Resultados

En la transformación:

$$x^1 = i X^1 ; \quad x^2 = X^2 ; \quad x^3 = X^3 ; \quad x^4 = X^4$$

El tensor H_{ij} se transforma como sigue:

$$h_{11} = -H_{11}$$
$$h_{1\alpha} = -i H_{1\alpha} \qquad \alpha, \beta = 2, 3, 4.$$
$$h_{\alpha 1} = h_{1\alpha}$$
$$h_{\alpha\beta} = H_{\alpha\beta}$$

$$dS^2 = (dX^1)^2 - (dX^2)^2 - (dX^3)^2 - (dX^4)^2$$

$$ds^2 = (dx^1)^2 + (dx^2)^2 + (dx^3)^2 + (dx^4)^2$$

$$\boxed{dS^2 = -ds^2}$$

$$\boxed{S = iS}$$

Transformación de las ecuaciones:

$$\frac{d^2X^i}{ds^2} = \Delta^{ip}\left[\frac{\partial H_{pj}}{\partial X^k} - \frac{\partial H_{jk}}{\partial X^p}\right]\frac{dX^j}{ds}\frac{dX^k}{ds}$$

$$\frac{d^2X^1}{ds^2} = \left(\frac{\partial H_{1j}}{\partial X^k} - \frac{\partial H_{jk}}{\partial X^1}\right)\frac{dX^j}{ds}\frac{dX^k}{ds}$$

$$\begin{aligned}
\frac{d^2X^1}{ds^2} = {} & \frac{\partial H_{11}}{\partial X^1}\frac{dX^1}{ds}\frac{dX^1}{ds} + \frac{\partial H_{1\alpha}}{\partial X^1}\frac{dX^\alpha}{ds}\frac{dX^1}{ds} \\
& + \frac{\partial H_{11}}{\partial X^\alpha}\frac{dX^1}{ds}\frac{dX^\alpha}{ds} + \frac{\partial H_{1\alpha}}{\partial X^\beta}\frac{dX^\alpha}{ds}\frac{dX^\beta}{ds} \\
& - \frac{\partial H_{11}}{\partial X^1}\frac{dX^1}{ds}\frac{dX^1}{ds} - \frac{\partial H_{1\alpha}}{\partial X^1}\frac{dX^\alpha}{ds}\frac{dX^1}{ds} \\
& - \frac{\partial H_{\alpha 1}}{\partial X^1}\frac{dX^1}{ds}\frac{dX^\alpha}{ds} - \frac{\partial H_{\alpha\beta}}{\partial X^1}\frac{dX^\alpha}{ds}\frac{dX^\beta}{ds}
\end{aligned}$$

$$\begin{aligned}
\frac{d^2x^1}{ds^2} = {} & -\frac{\partial h_{11}}{\partial x^1}\frac{dx^1}{ds}\frac{dx^1}{ds} - \frac{\partial h_{1\alpha}}{\partial x^1}\frac{dx^\alpha}{ds}\frac{dx^1}{ds} \\
& - \frac{\partial h_{11}}{\partial x^\alpha}\frac{dx^1}{ds}\frac{dx^\alpha}{ds} - \frac{\partial h_{1\alpha}}{\partial x^\beta}\frac{dx^\alpha}{ds}\frac{dx^\beta}{ds} \\
& + \frac{\partial h_{11}}{\partial x^1}\frac{dx^1}{ds}\frac{dx^1}{ds} + \frac{\partial h_{1\alpha}}{\partial x^1}\frac{dx^\alpha}{ds}\frac{dx^1}{ds} \\
& + \frac{\partial h_{\alpha 1}}{\partial x^1}\frac{dx^1}{ds}\frac{dx^\alpha}{ds} + \frac{\partial h_{\alpha\beta}}{\partial x^1}\frac{dx^\alpha}{ds}\frac{dx^\beta}{ds}
\end{aligned}$$

$$\frac{d^2x^1}{ds^2} = -\left(\frac{\partial h_{1j}}{\partial x^k} - \frac{\partial h_{jk}}{\partial x^1}\right)\frac{dx^j}{ds}\frac{dx^k}{ds}.$$

$$\frac{d^2X^\alpha}{ds^2} = -\left(\frac{\partial H_{\alpha j}}{\partial X^k} - \frac{\partial H_{jk}}{\partial X^\alpha}\right)\frac{dX^j}{ds}\frac{dX^k}{ds}$$

$$\begin{aligned}\frac{d^2X^\alpha}{ds^2} = \Bigg[&- \frac{\partial H_{\alpha 1}}{\partial X^1}\frac{dX^1}{ds}\frac{dX^1}{ds} - \frac{\partial H_{\alpha\beta}}{\partial X^1}\frac{dX^\beta}{ds}\frac{dX^1}{ds} \\ &- \frac{\partial H_{\alpha 1}}{\partial X^\beta}\frac{dX^1}{ds}\frac{dX^\beta}{ds} - \frac{\partial H_{\alpha\beta}}{\partial X^\gamma}\frac{dX^\beta}{ds}\frac{dX^\gamma}{ds} \\ &+ \frac{\partial H_{11}}{\partial X^\alpha}\frac{dX^1}{ds}\frac{dX^1}{ds} + \frac{\partial H_{1\beta}}{\partial X^\alpha}\frac{dX^1}{ds}\frac{dX^\beta}{ds} \\ &+ \frac{\partial H_{\beta 1}}{\partial X^\alpha}\frac{dX^\beta}{ds}\frac{dX^1}{ds} + \frac{\partial H_{\beta\gamma}}{\partial X^\alpha}\frac{dX^\beta}{ds}\frac{dX^\gamma}{ds}\Bigg]\end{aligned}$$

$$\begin{aligned}\frac{d^2x^\alpha}{ds^2} = \Bigg[&- \frac{\partial h_{\alpha 1}}{\partial x^1}\frac{dx^1}{ds}\frac{dx^1}{ds} - \frac{\partial h_{\alpha\beta}}{\partial x^1}\frac{dx^\beta}{ds}\frac{dx^1}{ds} \\ &- \frac{\partial h_{\alpha 1}}{\partial x^\beta}\frac{dx^1}{ds}\frac{dx^\beta}{ds} - \frac{\partial h_{\alpha\beta}}{\partial x^\gamma}\frac{dx^\beta}{ds}\frac{dx^\gamma}{ds} \\ &+ \frac{\partial h_{11}}{\partial x^\alpha}\frac{dx^1}{ds}\frac{dx^1}{ds} + \frac{\partial h_{1\beta}}{\partial x^\alpha}\frac{dx^1}{ds}\frac{dx^\beta}{ds} \\ &+ \frac{\partial h_{\beta 1}}{\partial x^\alpha}\frac{dx^\beta}{ds}\frac{dx^1}{ds} + \frac{\partial h_{\beta\gamma}}{\partial x^\alpha}\frac{dx^\beta}{ds}\frac{dx^\gamma}{ds}\Bigg]\end{aligned}$$

$$\frac{d^2x^\alpha}{ds^2} = -\left(\frac{\partial h_{\alpha j}}{\partial x^k} - \frac{\partial h_{jk}}{\partial x^\alpha}\right)\frac{dx^j}{ds}\frac{dx^k}{ds}.$$

Resultados def

Métrica pseudoeuclidiana.
Coordenadas reales
$dS^2 > 0$ intervalos temporaloides.

$$dS^2 = (dX^1)^2 - (dX^2)^2 - (dX^3)^2 - (dX^4)^2$$

$$dS^2 = \Delta_{ij}\, dX^i\, dX^j$$

Movimiento de una partícula en el campo gravitacional

$$\frac{d^2X^i}{dS^2} = \Delta^{ip}\left(\frac{\partial H_{pj}}{\partial X^k} - \frac{\partial H_{jk}}{\partial X^p}\right)\frac{dX^j}{dS}\,\frac{dX^k}{dS}$$

Rel

$i X^1$

X^2

X^3

X^4

$i S$

$dS^2 =$

H_{11}

$i H_{1\alpha}$

$i H_{\alpha 1}$

$- H_{\alpha\beta}$

def ...itivos.

Métrica euclidiana completa.
(completamente euclidiana).
coordenada temporal imaginaria.
$ds^2 > 0$ intervalo espacialoide.

$$ds^2 = (dx^1)^2 + (dx^2)^2 + (dx^3)^2 + (dx^4)^2$$

$$ds^2 = \delta_{ij}\, dx^i\, dx^j$$

Movimiento de una partícula en el campo gravitacional.

$$\frac{d^2x^i}{ds^2} = \left(\frac{\partial h_{ij}}{\partial x^k} - \frac{\partial h_{jk}}{\partial x^i}\right)\frac{dx^j}{ds}\frac{dx^k}{ds}.$$

Rel...aciones.

$iX^1 = x^1$

$X^2 = x^2$

$X^3 = x^3$

$X^4 = x^4$

$iS = s$

$dS^2 = -ds^2$

$H_{11} = h_{11}$

$iH_{1\alpha} = h_{1\alpha}$

$iH_{\alpha 1} = h_{\alpha 1}$

$-H_{\alpha\beta} = h_{\alpha\beta}$

$$\delta(ds)$$

$$ds^2 = dx^i\, dx^i$$

$$\delta(ds)^2 = 2\, ds\, \delta(ds) = 2\, dx^i\, d(\delta x^i)$$

$$\boxed{\delta ds = v^i\, d(\delta x^i)} \Longleftarrow$$

$$\delta v^i = \delta \frac{dx^i}{ds} = \frac{ds\, d(\delta x^i) - dx^i\, v^j\, d(\delta x^j)}{ds^2}$$

$$\boxed{\delta v^i = \frac{d}{ds}(\delta x^i) - v^i\left[v^j\, d(\delta x^j)\right]} \Longleftarrow\Longleftarrow$$

$$\mathcal{L}_0\, ds = A\, ds \qquad\qquad \mathcal{L}_0 = A$$

$$\delta(A\, ds)$$

$$\frac{\partial A}{\partial x^p}\delta x^p ds + A\, v^i\, d(\delta x^i)$$

$$\boxed{\delta \int A\, ds}$$

$$\frac{\partial A}{\partial x^p}\delta x^p ds - \frac{\partial A}{\partial x^j} v^j v^p \delta x^p ds$$

$$-A\, a^p\, \delta x^p\, ds$$

$$\frac{\partial A}{\partial x^p} - A a^p - \frac{\partial A}{\partial x^j} v^j v^p$$

$$\frac{\partial A}{\partial x^p} - A\left(\frac{\partial h_{pi}}{\partial x^j} - \frac{\partial h_{ij}}{\partial x^p}\right) v^i v^j - \frac{\partial A}{\partial x^j} v^j v^p$$

$$\left[\frac{\partial A}{\partial x^p}\delta_{ij} - A\frac{\partial h_{pi}}{\partial x^j} + A\frac{\partial h_{ij}}{\partial x^p} - \frac{\partial A}{\partial x^j}\delta_{pi}\right] v^i v^j$$

$$\delta\int A\,ds = \int\left[\frac{\partial A}{\partial x^p}\delta_{ij} - A\frac{\partial h_{pi}}{\partial x^j} + A\frac{\partial h_{ij}}{\partial x^p} - \frac{\partial A}{\partial x^j}\delta_{pi}\right] v^i v^j \delta x^p\, ds$$

$$\mathcal{L}_2 ds = A_{ij} v^i dx^j$$

$$\delta \mathcal{L}_2 ds = \Big[\frac{\partial A_{ij}}{\partial x^p} v^i v^j \delta x^p ds + A_{pj} v^j d(\delta x^p) - A_{ij} v^i v^j v^p d(\delta x^p) \Big] \Big[+ A_{ip} v^i d(\delta x^p) \Big]$$

$\delta \mathcal{L}_2 ds$ →

$$\delta \int \mathcal{L}_2 ds$$

$$\Big[\frac{\partial A_{ij}}{\partial x^p} v^i v^j - \frac{\partial A_{pj}}{\partial x^i} v^i v^j - A_{pj} a^j + \frac{\partial A_{ij}}{\partial x^k} v^i v^j v^k v^p + 2 A_{ij} a^i v^j v^p + A_{ij} v^i v^j a^p - \frac{\partial A_{ip}}{\partial x^j} v^i v^j - A_{ip} a^i \Big] \delta x^p ds$$

$$\left[\frac{\partial A_{ij}}{\partial x^p} - \frac{\partial A_{pj}}{\partial x^i} - A_{pk}\left(\frac{\partial h_{ki}}{\partial x^j} - \frac{\partial h_{ij}}{\partial x^k}\right) - \frac{\partial A_{ip}}{\partial x^j} - A_{kp}\left(\frac{\partial h_{ki}}{\partial x^j} - \frac{\partial h_{ij}}{\partial x^k}\right)\right] v^i v^j$$

$$\left.\begin{aligned} &\frac{\partial A}{\partial x^p}\delta_{ij} - A\frac{\partial h_{pi}}{\partial x^j} + A\frac{\partial h_{ij}}{\partial x^p} - \frac{\partial A}{\partial x^j}\delta_{pi} \\ &+ \frac{\partial A_{ij}}{\partial x^p} - \frac{\partial A_{pj}}{\partial x^i} - 2A_{pk}\left(\frac{\partial h_{ki}}{\partial x^j} - \frac{\partial h_{ij}}{\partial x^k}\right) \\ &- \frac{\partial A_{ip}}{\partial x^j} \end{aligned}\right\} = 0$$

$$\left(\frac{\partial A_{ij}}{\partial x^k} + 2A_{qj}\left(\frac{\partial h_{qi}}{\partial x^k} - \frac{\partial h_{ik}}{\partial x^q}\right)\right) v^i v^j v^k v^p$$

$$+ A_{ij} v^i v^j \left(\frac{\partial h_{pk}}{\partial x^l} - \frac{\partial h_{kl}}{\partial x^p}\right) v^k v^l$$

$$\left.\begin{aligned} &\frac{\partial A_{ij}}{\partial x^k}\delta_{pl} + 2A_{qj}\delta_{pl}\left(\frac{\partial h_{qi}}{\partial x^k} - \frac{\partial h_{ik}}{\partial x^q}\right) \\ &+ A_{ij}\left(\frac{\partial h_{pk}}{\partial x^l} - \frac{\partial h_{kl}}{\partial x^p}\right) \end{aligned}\right\} v^i v^j v^k v^l$$

Caso del espacio físico unidimensional

$$\frac{d^2x^1}{ds^2} = \left(\frac{\partial h_{1j}}{\partial x^k} - \frac{\partial h_{jk}}{\partial x^1}\right)\frac{dx^j}{ds}\frac{dx^k}{ds}$$

$$\frac{d^2x^1}{ds^2} = \left[\left(\frac{\partial h_{11}}{\partial x^2} - \frac{\partial h_{12}}{\partial x^1}\right)\frac{dx^1}{ds}\frac{dx^2}{ds} + \left(\frac{\partial h_{12}}{\partial x^1} - \frac{\partial h_{21}}{\partial x^1}\right)\frac{dx^2}{ds}\frac{dx^1}{ds} + \left(\frac{\partial h_{12}}{\partial x^2} - \frac{\partial h_{22}}{\partial x^1}\right)\frac{dx^2}{ds}\frac{dx^2}{ds}\right]$$

$$\frac{d^2x^1}{ds^2} = \left[\left(\frac{\partial h_{11}}{\partial x^2} - \frac{\partial h_{12}}{\partial x^1}\right)\frac{dx^1}{ds}\frac{dx^2}{ds} + \left(\frac{\partial h_{12}}{\partial x^2} - \frac{\partial h_{22}}{\partial x^1}\right)\frac{dx^2}{ds}\frac{dx^2}{ds}\right]$$

$x^1 \equiv \tau \quad ; \quad x^2 \equiv x$

$$\frac{d^2\tau}{ds^2} = \left[\left(\frac{\partial h_{11}}{\partial x} - \frac{\partial h_{12}}{\partial \tau}\right)\frac{d\tau}{ds}\frac{dx}{ds} + \left(\frac{\partial h_{12}}{\partial x} - \frac{\partial h_{22}}{\partial \tau}\right)\left(\frac{dx}{ds}\right)^2\right]$$

La Integral de Acción

La integral de acción para el caso de un campo generado por una ~~partícula~~ punto masa en movimiento ~~arbitrario~~ con cuadrivector Velocidad constante

μ ——— masa en gramos del punto masa

G ——— constante gravitacional (CGS)

$\hat{V}$ ——— cuadrivector velocidad del punto masa.

$\vec{V}$ ——— vector velocidad del punto masa.

V ——— velocidad del punto masa en cm/seg.

$$\hat{V} = \left(\frac{1}{\sqrt{1-\frac{V^2}{c^2}}} , \frac{\vec{V}}{\sqrt{1-\frac{V^2}{c^2}}} \right)$$

$$\hat{V} \cdot \hat{V} = 1$$

Coordenadas

x ——— abscisa en cm.

y ——— ordenada en cm

z ——— cota en cm

t ——— tiempo en seg.

$$ds^2 = c^2dt^2 - dx^2 - dy^2 - dz^2$$

$$\boxed{ds = c\sqrt{1 - \frac{v^2}{c^2}}\, dt}$$

(t, x, y, z) ——— acontecimiento en la línea de universo de la partícula explorada

(T, X, Y, Z) ——— acontecimiento retardado en la línea de universo

m ——— masa de la partícula exploradora en gramos.

$\hat{v}$ ——— cuadrivector velocidad de la partícula exploradora

$\vec{v}$ ——— vector velocidad de la partícula exploradora

v ——— velocidad de la partícula exploradora en cm/seg.

$$\hat{v} = \left(\frac{1}{\sqrt{1-\frac{v^2}{c^2}}} , \frac{\frac{\vec{V}}{c}}{\sqrt{1-\frac{v^2}{c^2}}} \right)$$

$$\hat{v} \cdot \hat{v} = 1 .$$

$$ds = c\sqrt{1-\frac{v^2}{c^2}}\,dt$$

$$ds^2 = c^2dt^2 - dx^2 - dy^2 - dz^2$$

La integral de acción es:

$$\mu = \frac{1}{2} mc \int e^{\frac{2Gu}{c^2(\hat{r}\cdot\hat{V})}} \left[2 - \frac{1}{(\hat{V}\cdot\hat{v})^2} \right] (\hat{V}\cdot\hat{v})\, ds$$

$$\hat{V} = \left(\frac{1}{\sqrt{1-\frac{V^2}{c^2}}} , \frac{\frac{\vec{V}}{c}}{\sqrt{1-\frac{V^2}{c^2}}} \right)$$

$$\hat{v} = \left(\frac{1}{\sqrt{1-\frac{v^2}{c^2}}} , \frac{\frac{\vec{v}}{c}}{\sqrt{1-\frac{v^2}{c^2}}} \right) .$$

$$\hat{V}\cdot\hat{v} = \frac{1 - \frac{\vec{V}\cdot\vec{v}}{c^2}}{\sqrt{1-\frac{v^2}{c^2}}\sqrt{1-\frac{V^2}{c^2}}}$$

$$ds = c\sqrt{1-\frac{v^2}{c^2}}\,dt$$

$$\left[2 - \frac{1}{(\hat{V}\cdot\hat{v})^2}\right](\hat{V}\cdot\hat{v})\,ds =$$

$$= \left[2 - \frac{\left(1-\frac{v^2}{c^2}\right)\left(1-\frac{V^2}{c^2}\right)}{\left(1-\frac{\vec{V}\cdot\vec{v}}{c^2}\right)^2}\right]\frac{\left(1-\frac{\vec{V}\cdot\vec{v}}{c^2}\right)\left[c\sqrt{1-\frac{v^2}{c^2}}\,dt\right]}{\sqrt{1-\frac{v^2}{c^2}}\sqrt{1-\frac{V^2}{c^2}}}$$

$$= \left[\frac{2\left(1-\frac{\vec{V}\cdot\vec{v}}{c^2}\right)}{\sqrt{1-\frac{V^2}{c^2}}} - \frac{\left(1-\frac{v^2}{c^2}\right)\sqrt{1-\frac{V^2}{c^2}}}{\left(1-\frac{\vec{V}\cdot\vec{v}}{c^2}\right)}\right]c\,dt$$

$$\frac{1}{2}mc\left[2-\frac{1}{(\hat{V}\cdot\hat{v})^2}\right](\hat{V}\cdot\hat{v})\,ds=$$

$$\frac{1}{2}mc^2\left[\frac{2\left(1-\frac{(\vec{V}\cdot\vec{v})}{c^2}\right)}{\sqrt{1-\frac{V^2}{c^2}}}-\frac{\left(1-\frac{v^2}{c^2}\right)\sqrt{1-\frac{V^2}{c^2}}}{\left(1-\frac{\vec{V}\cdot\vec{v}}{c^2}\right)}\right]dt$$

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$$\hat{r}=\left(c(t-T),\ x-X,\ y-Y,\ z-Z\right)$$

$$\hat{V}=\left[\frac{1}{\sqrt{1-\frac{V^2}{c^2}}},\ \frac{\vec{V}}{\sqrt{1-\frac{V^2}{c^2}}}\right]$$

$$\hat{r}=\left[c(t-T),\ \vec{r}\right]$$

$$(\hat{V}\cdot\hat{r})=\frac{c(t-T)-\vec{r}\cdot\frac{\vec{V}}{c}}{\sqrt{1-\frac{V^2}{c^2}}}$$

$$c(t-T)=r$$

$$\left(\hat{V}\cdot\hat{r}\right) = \frac{r - \frac{\vec{r}\cdot\vec{V}}{c}}{\sqrt{1-\frac{V^2}{c^2}}}$$

$$\left(\hat{V}\cdot\hat{r}\right) = \frac{r - \frac{\vec{r}\cdot\vec{V}}{c}}{\sqrt{1-\frac{V^2}{c^2}}}$$

$$e^{\frac{2G\mu\sqrt{1-\frac{V^2}{c^2}}}{c^2\left(r-\frac{\vec{r}\cdot\vec{V}}{c}\right)}}$$

$$e^{\frac{2G\mu}{c^2(\hat{r}\cdot\hat{V})}} = e^{\frac{2G\mu\sqrt{1-\frac{V^2}{c^2}}}{c^2\left(r-\frac{\vec{r}\cdot\vec{V}}{c}\right)}}$$

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$$\gamma = \frac{1}{2}mc^2 \int e^{\frac{2G\mu\sqrt{1-\frac{V^2}{c^2}}}{c^2\left(r-\frac{\vec{r}\cdot\vec{V}}{c}\right)}} \left[\quad \right] dt$$

$$\left[\frac{2\left(1-\frac{(\vec{V}\cdot\vec{v})}{c^2}\right)}{\sqrt{1-\frac{V^2}{c^2}}} - \frac{\left(1-\frac{v^2}{c^2}\right)\sqrt{1-\frac{V^2}{c^2}}}{\left(1-\frac{\vec{V}\cdot\vec{v}}{c^2}\right)} \right]$$

Para simplificar la notación hago $c=1$ (provisionalmente).

$$\frac{G\mu}{c^2} = M$$

$$\gamma = \frac{1}{2}m \int e^{\frac{2M\sqrt{1-V^2}}{r-\vec{r}\cdot\vec{V}}} \left[\frac{2\left(1-(\vec{V}\cdot\vec{v})\right)}{\sqrt{1-V^2}} - \frac{(1-v^2)\sqrt{1-V^2}}{(1-\vec{V}\cdot\vec{v})} \right] dt$$

$$\mathcal{R} = \frac{1}{r-\vec{r}\cdot\vec{V}}$$

$$\gamma = \frac{1}{2} m \int e^{2M\sqrt{1-V^2}\,\mathcal{R}} \left[\frac{2\left[1-(\vec{V}\cdot\vec{v})\right]}{\sqrt{1-V^2}} - \frac{(1-v^2)\sqrt{1-V^2}}{\left[1-(\vec{V}\cdot\vec{v})\right]} \right] dt$$

$$\mathcal{L} = e^{2M\sqrt{1-V^2}\,\mathcal{R}} \left[\frac{2\left[1-(\vec{V}\cdot\vec{v})\right]}{\sqrt{1-V^2}} - \frac{(1-v^2)\sqrt{1-V^2}}{\left[1-(\vec{V}\cdot\vec{v})\right]} \right]$$

Desarrollo en serie de $\mathcal{L}$ según potencias crecientes de c^{-1}

$$\sqrt{1-V^2} = \left[1 + \frac{1}{2}(-V^2) + \frac{(\frac{1}{2})(-\frac{1}{2})}{1\cdot 2}(-V^2)^2 + \frac{\frac{1}{2}(-\frac{1}{2})(-\frac{3}{2})}{1\cdot 2\cdot 3}(-V^2)^3 + \dots\right]$$

$$\sqrt{1-V^2} = \left[1 - \frac{1}{2}V^2 - \frac{1}{8}V^4 - \frac{1}{16}V^6 + \dots\right]$$

$$\left(\sqrt{1-V^2}\right)^2 = 1 - V^2$$

$$\left(\sqrt{1-V^2}\right)^3 = \left[1 + \frac{3}{2}(-V^2) + \frac{(\frac{3}{2})(\frac{1}{2})}{1\cdot 2}(-V^2)^2 + \frac{(\frac{3}{2})(\frac{1}{2})(-\frac{1}{2})}{1\cdot 2\cdot 3}(-V^2)^3 + \dots\right]$$

$$\left(\sqrt{1-V^2}\right)^3 = \left[1 - \frac{3}{2}V^2 + \frac{3}{8}V^4 + \frac{1}{8}V^6 + \dots\right]$$

$$e^{2M\sqrt{1-V^2}\mathcal{R}} = \left[1 + 2M\sqrt{1-V^2}\mathcal{R} + 2M^2(1-V^2)\mathcal{R}^2 + \frac{4}{3}M^3\left(\sqrt{1-V^2}\right)^3\mathcal{R}^3\right]$$

$$e^{2M\sqrt{1-V^2}\mathcal{R}} = \left[1 + 2M\mathcal{R} - \frac{1}{2}MV^2\mathcal{R} - \frac{1}{4}MV^4\mathcal{R} + 2M^2\mathcal{R}^2 - \frac{1}{2}M^2V^2\mathcal{R}^2 + \frac{4}{3}M^3\mathcal{R}^3\right]$$

comprobado

$$\sqrt{1-V^2} = 1 - \frac{1}{2}V^2 - \frac{1}{8}V^4 - \frac{1}{16}V^6 + \ldots\ldots$$

$$1 - 2(\vec{V}\cdot\vec{v})$$

$$\frac{2(1-\vec{V}\cdot\vec{v})}{\sqrt{1-V^2}} =$$

$$(1-V^2)^{-\frac{1}{2}} = 1 + \left(-\tfrac{1}{2}\right)(-V^2) + \frac{\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)}{1\cdot 2}(-V^2)^2 + \frac{\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)\left(-\frac{5}{2}\right)}{1\cdot 2\cdot 3}(-V^2)^3$$

$$(1-V^2)^{-\frac{1}{2}} = 1 + \frac{1}{2}V^2 + \frac{3}{8}V^4 + \frac{5}{16}V^6 + \dots$$

$$2(1-\vec{V}\cdot\vec{v}) = 2 - 2(\vec{V}\cdot\vec{v})$$

$$\frac{2\left[1-(\vec{V}\cdot\vec{v})\right]}{\sqrt{1-V^2}} =$$

$$2$$
$$+V^2 - 2(\vec{V}\cdot\vec{v})$$
$$+\frac{3}{4}V^4 - V^2(\vec{V}\cdot\vec{v})$$
$$+\frac{5}{8}V^6 - \frac{3}{4}V^4(\vec{V}\cdot\vec{v})$$

$$\sqrt{1-V^2} = 1 - \frac{1}{2}V^2 - \frac{1}{8}V^4 - \frac{1}{16}V^6 + \dots$$

$$(1-v^2) = 1 - v^2$$

$$1$$
$$-\frac{1}{2}V^2 - v^2$$
$$-\frac{1}{8}V^4 + \frac{1}{2}V^2v^2$$
$$-\frac{1}{16}V^6 + \frac{1}{8}V^4v^2$$

$$(1-v^2)\sqrt{1-V^2} = \left[1 + \left\{-\frac{1}{2}V^2 - v^2\right\} + \left\{-\frac{1}{8}V^4 + \frac{1}{2}V^2v^2\right\} + \left\{-\frac{1}{16}V^6 + \frac{1}{8}V^4v^2\right\} + \dots\right]$$

$$\frac{1}{1-(\vec{V}\cdot\vec{v})} = 1 + (\vec{V}\cdot\vec{v}) + (\vec{V}\cdot\vec{v})^2 + (\vec{V}\cdot\vec{v})^3 + \dots$$

$$\frac{(1-v^2)\sqrt{1-V^2}}{\{1-(\vec{V}\cdot\vec{v})\}} =$$

$$-1$$

$$+\frac{1}{2}V^2 + v^2 - (\vec{V}\cdot\vec{v})$$

$$+\frac{1}{8}V^4 - \frac{1}{2}V^2v^2 + \frac{1}{2}V^2(\vec{V}\cdot\vec{v}) + v^2(\vec{V}\cdot\vec{v}) - (\vec{V}\cdot\vec{v})^2$$

$$+\frac{1}{16}V^6 - \frac{1}{8}V^4v^2 + \frac{1}{8}V^4(\vec{V}\cdot\vec{v}) - \frac{1}{2}V^2v^2(\vec{V}\cdot\vec{v})$$

$$+\frac{1}{2}V^2(\vec{V}\cdot\vec{v})^2 + (\vec{V}\cdot\vec{v})^2v^2 - (\vec{V}\cdot\vec{v})^3$$

$$-\frac{(1-v^2)\sqrt{1-V^2}}{\{1-\vec{V}\cdot\vec{v}\}} = \begin{cases} -1 \\ +\left\{+\frac{1}{2}V^2+v^2-(\vec{V}\cdot\vec{v})\right\} \\ +\left\{+\frac{1}{8}V^4-\frac{1}{2}V^2v^2 \right. \\ \left. +\frac{1}{2}V^2(\vec{V}\cdot\vec{v})+v^2(\vec{V}\cdot\vec{v}) \right. \\ \left. -(\vec{V}\cdot\vec{v})^2 \right\} \\ +\left\{\frac{1}{16}V^6-\frac{1}{8}V^4v^2+\frac{1}{8}V^4(\vec{V}\cdot\vec{v}) \right. \\ \left. -\frac{1}{2}V^2v^2(\vec{V}\cdot\vec{v}) \right. \\ \left. +\frac{1}{2}V^2(\vec{V}\cdot\vec{v})^2+(\vec{V}\cdot\vec{v})^2v^2 \right. \\ \left. -(\vec{V}\cdot\vec{v})^3 \right\} \end{cases}$$

Comprobado

$$\left[\frac{1-(\vec{V}\cdot\vec{v})}{\sqrt{1-V^2}} - \frac{(1-v^2)\sqrt{1-V^2}}{\{1-(\vec{V}\cdot\vec{v})\}}\right] =$$

$$1^{0}$$

$$+\left\{\frac{3}{2}V^2 + v^2 - 3(\vec{V}\cdot\vec{v})\right\}$$

$$+\left\{\begin{array}{l}\frac{7}{8}V^4 - \frac{1}{2}V^2v^2 - \frac{1}{2}V^2(\vec{V}\cdot\vec{v}) \\ + v^2(\vec{V}\cdot\vec{v}) - (\vec{V}\cdot\vec{v})^2\end{array}\right\}$$

$$+\left\{\begin{array}{l}\frac{11}{16}V^6 - \frac{1}{8}V^4v^2 - \frac{5}{8}V^4(\vec{V}\cdot\vec{v}) \\ - \frac{1}{2}V^2v^2(\vec{V}\cdot\vec{v}) + \frac{1}{2}V^2(\vec{V}\cdot\vec{v})^2 \\ + (\vec{V}\cdot\vec{v})^2v^2 - (\vec{V}\cdot\vec{v})^3\end{array}\right\}$$

¡Comprobado!

$$+\frac{3}{2}M^2V^2\mathcal{R}^2+\frac{1}{2}M^2v^2\mathcal{R}^2-\frac{1}{2}M^2(\vec{V}\cdot\vec{v})\mathcal{R}^2$$

$$-\frac{1}{4}MV^4\mathcal{R}-\frac{1}{2}M^2V^2\mathcal{R}^2+\frac{1}{3}M^3\mathcal{R}^3$$

$$\mathcal{L} =$$

1

$$\frac{3}{2}V^2 + v^2 - 3(\vec{V}\cdot\vec{v}) + 2M\mathcal{R}$$

$$\frac{7}{8}V^4 - \frac{1}{2}V^2v^2 - \frac{1}{2}V^2(\vec{V}\cdot\vec{v}) + v^2(\vec{V}\cdot\vec{v}) - (\vec{V}\cdot\vec{v})^2 + \frac{3}{2}MV^2\mathcal{R} + 2Mv^2\mathcal{R} - 6M(\vec{V}\cdot\vec{v})\mathcal{R} - \frac{11}{12}MV^2\mathcal{R} + \frac{7}{2}M^2\mathcal{R}^2$$

dos

$$+\frac{11}{16}V^6 - \frac{1}{8}V^4v^2 - \frac{5}{8}V^4(\vec{V}\cdot\vec{v}) - \frac{1}{2}V^2v^2(\vec{V}\cdot\vec{v}) + \frac{1}{2}V^2(\vec{V}\cdot\vec{v})^2 + (\vec{V}\cdot\vec{v})^2v^2 - (\vec{V}\cdot\vec{v})^3 + \frac{7}{4}MV^4\mathcal{R} - \frac{11}{12}MV^2v^2\mathcal{R} - \frac{1}{2}MV^2(\vec{V}\cdot\vec{v})\mathcal{R} + 2M(\vec{V}\cdot\vec{v})v^2\mathcal{R} - 2M(\vec{V}\cdot\vec{v})^2\mathcal{R} - \frac{3}{2}MV^4\mathcal{R} - \frac{1}{2}MV^2v^2\mathcal{R} + \frac{3}{2}MV^2(\vec{V}\cdot\vec{v})\mathcal{R}$$

$$\mathcal{L} =$$

1

$$\frac{3}{2}V^2 + v^2 - 3(\vec{V}\cdot\vec{v}) + 2M\mathcal{R}$$

$$+\frac{7}{8}V^4 - \frac{1}{2}V^2v^2 - \frac{1}{2}V^2(\vec{V}\cdot\vec{v}) + v^2(\vec{V}\cdot\vec{v})$$

$$-(\vec{V}\cdot\vec{v})^2 + 2MV^2\mathcal{R} + 2Mv^2\mathcal{R} - 6M(\vec{V}\cdot\vec{v})\mathcal{R}$$

$$+ 2M^2\mathcal{R}$$

$$+\frac{11}{16}V^6 - \frac{1}{8}V^4v^2 - \frac{5}{8}V^4(\vec{V}\cdot\vec{v})$$

$$-\frac{1}{2}V^2v^2(\vec{V}\cdot\vec{v}) + \frac{1}{2}V^2(\vec{V}\cdot\vec{v})^2$$

$$+(\vec{V}\cdot\vec{v})^2v^2 - (\vec{V}\cdot\vec{v})^3 - 2MV^2v^2\mathcal{R}$$

$$+2MV^2(\vec{V}\cdot\vec{v})\mathcal{R} + 2M(\vec{V}\cdot\vec{v})v^2\mathcal{R}$$

$$-2M(\vec{V}\cdot\vec{v})^2\mathcal{R} + M^2V^2\mathcal{R}^2$$

$$+2M^2v^2\mathcal{R}^2 - 6M^2(\vec{V}\cdot\vec{v})\mathcal{R}^2$$

$$+\frac{4}{3}M^3\mathcal{R}^3$$

$\mathcal{L} =$

1

$$+\frac{3}{2}V^2 - 3(\vec{V}\cdot\vec{v}) + v^2 + 2M\mathcal{R}$$

$$+\frac{7}{8}V^4 - \frac{1}{2}V^2(\vec{V}\cdot\vec{v}) - (\vec{V}\cdot\vec{v})^2 - \frac{1}{2}V^2v^2 + (\vec{V}\cdot\vec{v})v^2$$
$$+3MV^2\mathcal{R} - 6M(\vec{V}\cdot\vec{v})\mathcal{R} + 2Mv^2\mathcal{R}$$
$$-MV^2\mathcal{R} + 2M^2\mathcal{R}^2$$

$$+\frac{11}{16}V^6 - \frac{5}{8}V^4(\vec{V}\cdot\vec{v}) + \frac{1}{2}V^2(\vec{V}\cdot\vec{v})^2 - (\vec{V}\cdot\vec{v})^3$$
$$-\frac{1}{8}V^4v^2 - \frac{1}{2}V^2(\vec{V}\cdot\vec{v})v^2 + (\vec{V}\cdot\vec{v})^2v^2$$
$$+\frac{7}{4}MV^4\mathcal{R} - MV^2(\vec{V}\cdot\vec{v})\mathcal{R} - 2M(\vec{V}\cdot\vec{v})^2\mathcal{R}$$
$$+2M(\vec{V}\cdot\vec{v})v^2\mathcal{R}$$
$$-\frac{3}{2}MV^4\mathcal{R} + 3MV^2(\vec{V}\cdot\vec{v})\mathcal{R} - MV^2v^2\mathcal{R}$$
$$+3M^2V^2\mathcal{R}^2 - 6M^2(\vec{V}\cdot\vec{v})\mathcal{R}^2 + 2M^2v^2\mathcal{R}^2$$
$$-\frac{1}{4}MV^4\mathcal{R} - 2M^2V^2\mathcal{R}^2 + \frac{4}{3}M^3\mathcal{R}^3$$

$$\mathcal{L} =$$

1

$$+\frac{3}{2}V^2 - 3(\vec{V}\cdot\vec{v}) + v^2 + 2M\mathcal{R}$$

$$+\frac{7}{8}V^4 - \frac{1}{2}V^2(\vec{V}\cdot\vec{v}) - (\vec{V}\cdot\vec{v})^2 - \frac{1}{2}Vv^2 + (\vec{V}\cdot\vec{v})v^2$$
$$+2MV^2\mathcal{R} - 6M(\vec{V}\cdot\vec{v})\mathcal{R} + 2Mv^2\mathcal{R}$$
$$+2M^2\mathcal{R}^2$$

$$+\frac{11}{16}V^6 - \frac{5}{8}V^4(\vec{V}\cdot\vec{v}) + \frac{1}{2}V^2(\vec{V}\cdot\vec{v})^2 - (\vec{V}\cdot\vec{v})^3$$
$$-\frac{1}{8}V^4v^2 - \frac{1}{2}V^2(\vec{V}\cdot\vec{v})v^2 + (\vec{V}\cdot\vec{v})^2v^2$$
$$+2MV^2(\vec{V}\cdot\vec{v})\mathcal{R} - 2M(\vec{V}\cdot\vec{v})^2\mathcal{R}$$
$$+2M(\vec{V}\cdot\vec{v})v^2\mathcal{R} - MV^2v^2\mathcal{R}$$
$$+M^2V^2\mathcal{R}^2 - 6M^2(\vec{V}\cdot\vec{v})\mathcal{R}^2$$
$$+2M^2v^2\mathcal{R}^2 + \frac{4}{3}M^3\mathcal{R}^3$$

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Desarrollo de $\vec{a}$ en función de $\vec{r}$, $\vec{V}$, $\vec{A}$, y $\vec{v}$. (hasta términos de 6º grado en c^{-1})

Términos en $\vec{r}$

Grado en c^{-1}

2 — $-M\mathcal{R}^3$

4 — $-\frac{1}{2}MV^2\mathcal{R}^3 - Mv^2\mathcal{R}^3 + 4M(\vec{V}\cdot\vec{v})\mathcal{R}^3$
$-M(\vec{A}\cdot\vec{r})\mathcal{R}^3$

5 — $-3M(\vec{V}\cdot\vec{A})\mathcal{R}^2 + 4M(\vec{A}\cdot\vec{v})\mathcal{R}^2$ $\vec{r}$

6 — $+\frac{5}{8}MV^4\mathcal{R}^3 + \frac{3}{2}MV^2v^2\mathcal{R}^3$
$-2MV^2(\vec{V}\cdot\vec{v})\mathcal{R}^3 - 2M(\vec{V}\cdot\vec{v})^2\mathcal{R}^3$
$-\frac{3}{2}MV^2(\vec{A}\cdot\vec{r})\mathcal{R}^3 - M(\vec{A}\cdot\vec{r})v^2\mathcal{R}^3$
$+4M(\vec{V}\cdot\vec{v})(\vec{A}\cdot\vec{r})\mathcal{R}^3$

¡Comprobado!

Desarrollo de $\vec{a}$ en función de $\vec{r}$, $\vec{V}$, $\vec{A}$ y $\vec{v}$. (Hasta términos de 6. grado en c^{-1})

Términos en $\vec{V}$

$$-MR^2 + 2Mr\mathcal{R}^3$$

$$-2M(\vec{r}\cdot\vec{v})\mathcal{R}^3$$

$$+\tfrac{1}{2}MV^2\mathcal{R}^2 + Mv^2\mathcal{R}^2$$

$$-MV^2 r\mathcal{R}^3 - 2M(\vec{V}\cdot\vec{v})r\mathcal{R}^3 \quad \vec{V}$$

$$+2M(\vec{A}\cdot\vec{r})r\mathcal{R}^3 - 2M(\vec{A}\cdot\vec{v})r\mathcal{R}^3$$

$$+MV^2(\vec{r}\cdot\vec{v})\mathcal{R}^3$$

$$+2M(\vec{V}\cdot\vec{v})(\vec{r}\cdot\vec{v})\mathcal{R}^3$$

$$+2M(\vec{V}\cdot\vec{A})r\mathcal{R}^2$$

$$-2M(\vec{A}\cdot\vec{r})(\vec{r}\cdot\vec{v})\mathcal{R}^3$$

ado en c^{-1}

3

4

5

6

correcto

correcto

Desarrollo de $\vec{a}$ en función de $\vec{r}$, $\vec{V}$, $\vec{A}$ y $\vec{F}$ (hasta términos de 6° grado en c^{-1})

Términos en $\vec{A}$.

en c^{-1}

4 $\quad 2Mr\mathcal{R}^2$

5 $\quad -2M(\vec{r}\cdot\vec{v})\mathcal{R}^2$

6 $\quad +MV^2r\mathcal{R}^2 - 2M(\vec{V}\cdot\vec{v})r\mathcal{R}^2$

$\rightarrow \vec{A}$

Desarrollo de $\vec{a}$ en función de $\vec{r}, \vec{V}$, $\vec{A}$ y $\vec{v}$. (Hasta términos de 6. grado en c^{-1}).

Términos en $\vec{v}$

$$+M\mathcal{R}^2\vec{v}$$

$$-Mr\mathcal{R}^3\vec{v}$$

$$+2M(\vec{r}\cdot\vec{v})\mathcal{R}^3\vec{v}$$

$$\div\frac{1}{2}MV^2\mathcal{R}^2\vec{v} - Mv^2\mathcal{R}^2\vec{v}$$

$$-M(\vec{A}\cdot\vec{r})r\cdot\mathcal{R}^3\vec{v}$$

$$+\frac{3}{2}MV^2r\mathcal{R}^3\vec{v} + Mv^2r\mathcal{R}^3\vec{v}$$

$$-2M(\vec{V}\cdot\vec{v})r\mathcal{R}^3\vec{v} - 2M(\vec{V}\cdot\vec{v})(\vec{r}\cdot\vec{v})\mathcal{R}^3\vec{v}$$

$$+M(\vec{V}\cdot\vec{A})r\mathcal{R}^2\vec{v} - 2M(\vec{A}\cdot\vec{v})r\mathcal{R}^2\vec{v}$$

$$-MV^2(\vec{r}\cdot\vec{v})\mathcal{R}^3\vec{v} + 2M(\vec{A}\cdot\vec{r})(\vec{r}\cdot\vec{v})\mathcal{R}^3\vec{v}$$

$(\vec{v}\cdot\vec{a})$ hasta términos de 6 grado en c^{-1}

Grado en c^{-1}

3 $-M(\vec{r}\cdot\vec{v})\mathcal{R}^3$

$-M(\vec{V}\cdot\vec{v})\mathcal{R}^2 + 2M(\vec{V}\cdot\vec{v})r\mathcal{R}^3$

4 ~~$+2M(\vec{A}\cdot\vec{r})\mathcal{R}^2$~~

$+Mv^2\mathcal{R}^2 - Mrv^2\mathcal{R}^3$

$-\frac{1}{2}MV^2(\vec{r}\cdot\vec{v})\mathcal{R}^3 - M(\vec{r}\cdot\vec{v})v^2\mathcal{R}^3$

5 $+4M(\vec{V}\cdot\vec{v})(\vec{r}\cdot\vec{v})\mathcal{R}^3 - M(\vec{A}\cdot\vec{r})(\vec{r}\cdot\vec{v})\mathcal{R}^3$

$-2M(\vec{V}\cdot\vec{v})(\vec{r}\cdot\vec{v})\mathcal{R}^3 + 2M(\vec{A}\cdot\vec{v})r\mathcal{R}^2$

$+2M(\vec{r}\cdot\vec{v})v^2\mathcal{R}^3$

$-3M(\vec{V}\cdot\vec{A})(\vec{r}\cdot\vec{v})\mathcal{R}^2 + 4M(\vec{A}\cdot\vec{v})(\vec{r}\cdot\vec{v})\mathcal{R}^2$

6 $+\frac{1}{2}MV^2(\vec{V}\cdot\vec{v})\mathcal{R}^2 + M(\vec{V}\cdot\vec{v})v^2\mathcal{R}^2$

$-MV^2(\vec{V}\cdot\vec{v})r\mathcal{R}^3 - 2M(\vec{V}\cdot\vec{v})^2r\mathcal{R}^3$

$+2M(\vec{V}\cdot\vec{v})(\vec{A}\cdot\vec{r})r\mathcal{R}^3 - 2M(\vec{V}\cdot\vec{v})(\vec{A}\cdot\vec{v})r\mathcal{R}^3$

$-2M(\vec{A}\cdot\vec{v})(\vec{r}\cdot\vec{v})\mathcal{R}^2$

$-\frac{1}{2}MV^2v^2\mathcal{R}^2 - Mv^4\mathcal{R}^2$

$-M(\vec{A}\cdot\vec{r})rv^2\mathcal{R}^3 + \frac{3}{2}MV^2rv^2\mathcal{R}^3$

$+Mrv^4\mathcal{R}^3 - 2M(\vec{V}\cdot\vec{v})rv^2\mathcal{R}^3$

~~$+M(\vec{A}\cdot\vec{v})(\vec{r}\cdot\vec{v})\mathcal{R}^3$~~

$(\vec{v}\cdot\vec{a})$ hasta términos de 6º grado en c^{-1}

rado en c^{-1}

3

$$-M(\vec{r}\cdot\vec{v})\mathcal{R}^3$$

4

$$-M(\vec{V}\cdot\vec{v})\mathcal{R}^2 + 2M(\vec{V}\cdot\vec{v})r\mathcal{R}^3$$
$$+Mv^2\mathcal{R}^2 - Mrv^2\mathcal{R}^3$$

correcto →

$$-\tfrac{1}{2}MV^2(\vec{r}\cdot\vec{v})\mathcal{R}^3 + M(\vec{r}\cdot\vec{v})v^2\mathcal{R}^3$$

5

$$+2M(\vec{V}\cdot\vec{v})(\vec{r}\cdot\vec{v})\mathcal{R}^3 + 2M(\vec{A}\cdot\vec{v})r\mathcal{R}^2$$
$$-M(\vec{A}\cdot\vec{r})(\vec{r}\cdot\vec{v})\mathcal{R}^3$$

6

$$-3M(\vec{V}\cdot\vec{A})(\vec{r}\cdot\vec{v})\mathcal{R}^2 + 2M(\vec{A}\cdot\vec{v})(\vec{r}\cdot\vec{v})\mathcal{R}^2$$
$$+\tfrac{1}{2}MV^2(\vec{V}\cdot\vec{v})\mathcal{R}^2 + M(\vec{V}\cdot\vec{v})v^2\mathcal{R}^2$$
$$-MV^2(\vec{V}\cdot\vec{v})r\mathcal{R}^3 - 2M(\vec{V}\cdot\vec{v})^2r\mathcal{R}^3$$
$$+2M(\vec{V}\cdot\vec{v})(\vec{A}\cdot\vec{r})r\mathcal{R}^3 \cancel{-2M(\vec{V}\cdot\vec{v})(\vec{A}\cdot\vec{v})r\mathcal{R}^3}$$
$$-\tfrac{1}{2}MV^2v^2\mathcal{R}^2 - Mv^4\mathcal{R}^2$$
$$-M(\vec{A}\cdot\vec{r})rv^2\mathcal{R}^3 + \tfrac{3}{2}MV^2rv^2\mathcal{R}^3$$
$$+Mrv^4\mathcal{R}^3 - 2M(\vec{V}\cdot\vec{v})rv^2\mathcal{R}^3$$

Comprobado con cuaderno 101 pág. 212

$(\vec{v}\cdot\vec{a})$ hasta términos de 6. grado en c^{-1} (Comprobación pág. 212 cuaderno 101)

Grado en c^{-1}

3

$$-M(\vec{r}\cdot\vec{v})\mathcal{R}^3$$

4

$$+Mv^2\mathcal{R}^2 - M(\vec{V}\cdot\vec{v})\mathcal{R}^2 - Mrv^2\mathcal{R}^3$$
$$+2M(\vec{V}\cdot\vec{v})r\mathcal{R}^3$$

$$+2M(\vec{A}\cdot\vec{v})r\mathcal{R}^2 - \frac{3}{2}MV^2(\vec{r}\cdot\vec{v})\mathcal{R}^3$$
$$-Mv^2(\vec{r}\cdot\vec{v})\mathcal{R}^3 + MV^2(\vec{r}\cdot\vec{v})\mathcal{R}^3 + 2M(\vec{r}\cdot\vec{v})v^2\mathcal{R}^3$$

tres

5

$$+2M(\vec{V}\cdot\vec{v})(\vec{r}\cdot\vec{v})\mathcal{R}^3 + M(\vec{A}\cdot\vec{r})(\vec{r}\cdot\vec{v})\mathcal{R}^3$$
$$-2M(\vec{A}\cdot\vec{r})(\vec{r}\cdot\vec{v})\mathcal{R}^3$$

$$-\frac{1}{2}MV^2v^2\mathcal{R}^2 - Mv^4\mathcal{R}^2$$
$$+\frac{1}{2}MV^2(\vec{V}\cdot\vec{v})\mathcal{R}^2 + M(\vec{V}\cdot\vec{v})v^2\mathcal{R}^2$$

6

$$-M(\vec{V}\cdot\vec{A})(\vec{r}\cdot\vec{v})\mathcal{R}^2 + 2M(\vec{A}\cdot\vec{v})(\vec{r}\cdot\vec{v})\mathcal{R}^2$$
$$-2M(\vec{V}\cdot\vec{A})(\vec{r}\cdot\vec{v})\mathcal{R}^2 + \frac{3}{2}MV^2rv^2\mathcal{R}^3$$
$$+Mrv^4\mathcal{R}^3 - MV^2(\vec{V}\cdot\vec{v})r\mathcal{R}^3$$
$$-2M(\vec{V}\cdot\vec{v})v^2\mathcal{R}^3 - 2M(\vec{V}\cdot\vec{v})^2r\mathcal{R}^3$$
$$-M(\vec{A}\cdot\vec{r})rv^2\mathcal{R}^3 + 2M(\vec{V}\cdot\vec{v})(\vec{A}\cdot\vec{r})r\mathcal{R}^3$$

Grado en c^{-1}

3

4

correcto →

5

$(\vec{v} \cdot \vec{a}) =$

$(\vec{v} \cdot \vec{a})$ hasta en c^{-1}

$-M(\vec{F} \cdot \vec{v})$

$-M(\vec{V} \cdot \vec{v}) R$

$+M v^2 R^2$

$-\frac{1}{2} M V^2 (\vec{F}$

$+2M(\vec{V} \cdot \vec{v})$

$-M(\vec{R} \cdot \vec{r})$

$-3M(\vec{V} \cdot \vec{R}$

$+\frac{1}{2} M V^2$

$-M V^2 (\vec{V}$

$+2M(\vec{V} \cdot$

$-M v^2 R$

$(\vec{v}\cdot\vec{a})$ hasta términos de 6. grado en c^{-1}

Grado en c^{-1}

3 $-M(\vec{r}\cdot\vec{v})\mathcal{R}^3$

$-M(\vec{v}\cdot\vec{v})\mathcal{R}^2 + 2M(\vec{v}\cdot\vec{v})r\mathcal{R}^3$

4 $+Mv^2\mathcal{R}^2 - Mrv^2\mathcal{R}^3$

correcto → $-\frac{1}{2}MV^2(\vec{r}\cdot\vec{v})\mathcal{R}^3 + M(\vec{r}\cdot\vec{v})v^2\mathcal{R}^3$

5 $+2M(\vec{v}\cdot\vec{v})(\vec{r}\cdot\vec{v})\mathcal{R}^3 + 2M(\vec{A}\cdot\vec{v})r\mathcal{R}^2$

$-M(\vec{A}\cdot\vec{r})(\vec{r}\cdot\vec{v})\mathcal{R}^3$

$-3M(\vec{v}\cdot\vec{A})(\vec{r}\cdot\vec{v})\mathcal{R}^2 + 2M(\vec{A}\cdot\vec{v})(\vec{r}\cdot\vec{v})\mathcal{R}^2$

$+\frac{1}{2}MV^2(\vec{v}\cdot\vec{v})\mathcal{R}^2 + M(\vec{v}\cdot\vec{v})v^2\mathcal{R}^2$

$-MV^2(\vec{v}\cdot\vec{v})r\mathcal{R}^3 - 2M(\vec{v}\cdot\vec{v})^2r\mathcal{R}^3$

$+2M(\vec{v}\cdot\vec{v})(\vec{A}\cdot\vec{r})r\mathcal{R}^3 - \frac{1}{2}MV^2v^2\mathcal{R}^2$

$-Mv^4\mathcal{R}^2 - M(\vec{A}\cdot\vec{r})rv^2\mathcal{R}^3$

recto → $+\frac{3}{2}MV^2rv^2\mathcal{R}^3$ $+Mrv^4\mathcal{R}^3$

$-2M(\vec{v}\cdot\vec{v})v^2\mathcal{R}^3$

Comprobado con cuaderno 101 pág. 212

Grado en c^{-1}

$(\vec{V}\cdot\vec{a})$ hasta términos de 6. grado en c^{-1}:

3 $-M(\vec{V}\cdot\vec{r})\mathcal{R}^3$

4 $-MV^2\mathcal{R}^2+2MV^2r\mathcal{R}^3+M(\vec{V}\cdot\vec{v})\mathcal{R}^2$
$-M(\vec{V}\cdot\vec{v})r\mathcal{R}^3$

5 $-2MV^2(\vec{r}\cdot\vec{v})\mathcal{R}^3+2M(\vec{V}\cdot\vec{A})r\mathcal{R}^2$
$-2M(\vec{V}\cdot\vec{v})r\mathcal{R}^3$
$-3M(\vec{V}\cdot\vec{A})(\vec{V}\cdot\vec{r})\mathcal{R}^2+4M(\vec{V}\cdot\vec{r})(\vec{A}\cdot\vec{v})\mathcal{R}^2$
$+\frac{1}{2}MV^4\mathcal{R}^2+MV^2v^2\mathcal{R}^2-MV^4r\mathcal{R}^3$
$-2MV^2(\vec{V}\cdot\vec{v})r\mathcal{R}^3+2MV^2(\vec{A}\cdot\vec{r})r\mathcal{R}^3$
$-2M(\vec{V}\cdot\vec{A})(\vec{r}\cdot\vec{v})\mathcal{R}^2-\frac{1}{2}MV^2(\vec{V}\cdot\vec{v})\mathcal{R}^2$
6 $-M(\vec{V}\cdot\vec{v})v^2\mathcal{R}^2-M(\vec{V}\cdot\vec{v})(\vec{A}\cdot\vec{r})r\mathcal{R}^3$
$+\frac{3}{2}MV^2(\vec{V}\cdot\vec{v})r\mathcal{R}^3+M(\vec{V}\cdot\vec{v})v^2r\mathcal{R}^3$
$-2M(\vec{V}\cdot\vec{v})^2r\mathcal{R}^3$

correctos

#

Grado en c^{-1}

3

4

5

$(\vec{V} \cdot \vec{a}) =$

6

$(\vec{V} \cdot \vec{a})$ hasta

$-M(\vec{V} \cdot \vec{r})$

$-MV^2 R^2 +$

$-M(\vec{V} \cdot \vec{v})$

$\div \frac{1}{2} MV^2 (\vec{V}$

$+4M(\vec{V} \cdot \vec{r})$

$-2MV^2 (\vec{r} \cdot$

$+2M(\vec{V} \cdot \vec{v})$

$-3M(\vec{V} \cdot \vec{N}$

$+\frac{1}{2} MV$

$-\frac{1}{2} MV^2$

$-2M(\vec{V} \cdot \vec{N}$

$-M(\vec{V} \cdot \vec{v})$

Grado en c^{-1} | $(\vec{V}\cdot\vec{a})$ hasta términos de 6. grado en c^{-1}

3 | $-M(\vec{V}\cdot\vec{r})\mathcal{R}^3$

4 | $-MV^2\mathcal{R}^2 + 2MV^2 r\mathcal{R}^3 + M(\vec{V}\cdot\vec{v})\mathcal{R}^2$
$-M(\vec{V}\cdot\vec{v})r\mathcal{R}^3$

$-\frac{1}{2}MV^2(\vec{V}\cdot\vec{r})\mathcal{R}^3 - M(\vec{V}\cdot\vec{r})v^2\mathcal{R}^3$

5 | $+4M(\vec{V}\cdot\vec{r})(\vec{V}\cdot\vec{v})\mathcal{R}^3 - M(\vec{V}\cdot\vec{r})(\vec{A}\cdot\vec{r})\mathcal{R}^3$
$-2MV^2(\vec{r}\cdot\vec{v})\mathcal{R}^3 + 2M(\vec{V}\cdot\vec{A})r\mathcal{R}^2$
$+2M(\vec{V}\cdot\vec{v})(\vec{r}\cdot\vec{v})\mathcal{R}^3$

6 | $-3M(\vec{V}\cdot\vec{A})(\vec{V}\cdot\vec{r})\mathcal{R}^2 + 4M(\vec{V}\cdot\vec{r})(\vec{A}\cdot\vec{v})\mathcal{R}^2$
$+\frac{1}{2}MV^4\mathcal{R}^2 + MV^2v^2\mathcal{R}^2 - MV^4 r\mathcal{R}^3$
$-\frac{1}{2}MV^2(\vec{V}\cdot\vec{v})r\mathcal{R}^3 + 2MV^2(\vec{A}\cdot\vec{r})r\mathcal{R}^3$
$-2M(\vec{V}\cdot\vec{A})(\vec{r}\cdot\vec{v})\mathcal{R}^2 - \frac{1}{2}MV^2(\vec{V}\cdot\vec{v})\mathcal{R}^2$
$-M(\vec{V}\cdot\vec{v})v^2\mathcal{R}^2 - M(\vec{V}\cdot\vec{v})(\vec{A}\cdot\vec{r})r\mathcal{R}^3$
$+M(\vec{V}\cdot\vec{v})v^2 r\mathcal{R}^3 - 2M(\vec{V}\cdot\vec{v})^2 r\mathcal{R}^3$

¡Comprobado!

Grado en $\vec{c}$

$$\frac{\partial \mathcal{L}}{\partial \vec{r}} =$$

1 $\quad -3\vec{V} + 2\vec{v}$

3 $\quad -\frac{1}{2}V^2\vec{V} - 2(\vec{V}\cdot\vec{v})\vec{V} - V^2\vec{v} + v^2\vec{V} + 2(\vec{V}\cdot\vec{v})\vec{v}$
$\quad - 6M\mathcal{R}\vec{V} + 4M\mathcal{R}\vec{v}$

5 $\quad -\frac{5}{8}V^4\vec{V} + V^2(\vec{V}\cdot\vec{v})\vec{V} - 3(\vec{V}\cdot\vec{v})^2\vec{V}$
$\quad -\frac{1}{4}V^4\vec{v} - \frac{1}{2}V^2v^2\vec{V} - V^2(\vec{V}\cdot\vec{v})\vec{v}$
$\quad + 2(\vec{V}\cdot\vec{v})v^2\vec{V} + 2(\vec{V}\cdot\vec{v})^2\vec{v} + 2MV^2\mathcal{R}\vec{V}$
$\quad -4M(\vec{V}\cdot\vec{v})\mathcal{R}\vec{V} + 2Mv^2\mathcal{R}\vec{V}$
$\quad + 4M(\vec{V}\cdot\vec{v})\mathcal{R}\vec{v} - 2MV^2\mathcal{R}\vec{v}$
$\quad + 6M^2\mathcal{R}^2\vec{V} + 4M^2\mathcal{R}^2\vec{v}.$

Comprobado

$$\frac{\partial \vec{V}}{\partial t} = \frac{r}{r - \vec{r}\cdot\vec{V}}\vec{A} = r\mathcal{R}\vec{A}$$

$$(\vec{v}\cdot\nabla)\vec{V} = -\frac{(\vec{r}\cdot\vec{v})}{(r - \vec{r}\cdot\vec{V})}\vec{A} = -(\vec{r}\cdot\vec{v})\mathcal{R}\vec{A}$$

$$\boxed{\frac{d\vec{V}}{dt} = r\dot{\mathcal{R}}\vec{A} - (\vec{r}\cdot\vec{v})\dot{\mathcal{R}}\vec{A}} \longleftarrow$$

$$-3\frac{d\vec{V}}{dt} = -3r\mathcal{R}\vec{A} + 3(\vec{r}\cdot\vec{v})\mathcal{R}\vec{A}$$

(2) (3)

Grado en c^{-1}

$$-3\frac{d\vec{V}}{dt} = \begin{bmatrix} -3r\mathcal{R} \\ +3(\vec{r}\cdot\vec{v})\mathcal{R} \end{bmatrix}\vec{A} \qquad \begin{matrix} 2 \\ 3 \end{matrix}$$

Grado en ϵ^{-1}	$\vec{r}$	$\vec{V}$
2	$-2MR^3$	
3		$-2MR^2$ $+4M\vec{r}\cdot\vec{V}R^3$
4	$-MV^2R^3$ $-2Mv^2R^3$ $+8M(\vec{V}\cdot\vec{v})R^3$ $-2M(\vec{A}\cdot\vec{r})R^3$	$-4M(\vec{r}\cdot\vec{v})R^3$
5	$-6M(\vec{V}\cdot\vec{A})R^2$ $+8M(\vec{A}\cdot\vec{v})R^2$	$+MV^2R^2$ $+2Mv^2R^2$ $-2MV^2rR^3$ $-4M(\vec{V}\cdot\vec{v})rR^3$ $+4M(\vec{A}\cdot\vec{r})rR^3$
6	$+\frac{5}{4}MV^4R^3$ $+3MV^2v^2R^3$ $-4MV^2(\vec{V}\cdot\vec{v})R^3$ $-4M(\vec{V}\cdot\vec{v})^2R^3$ $-3MV^2(\vec{A}\cdot\vec{r})R^3$	$-4M(\vec{A}\cdot\vec{v})rR^3$ $+2MV^2(\vec{r}\cdot\vec{v})R^3$ $+4M(\vec{V}\cdot\vec{v})(\vec{r}\cdot\vec{v})R^3$ $+4M(\vec{V}\cdot\vec{A})rR^2$ $-4M(\vec{A}\cdot\vec{r})(\vec{r}\cdot\vec{v})R^3$

Tres

$2\vec{a}$ =

$2\vec{a} =$

	$\vec{A}$	$\vec{V}$
		$+2M\mathcal{R}^2$
		$-2Mr\mathcal{R}^3$
$+4Mr\mathcal{R}^2$		$+4M(\vec{r}\cdot\vec{v})\mathcal{R}^3$
$-4M(\vec{r}\cdot\vec{v})\mathcal{R}^2$		$-MV^2\mathcal{R}^2$
		$-2Mv^2\mathcal{R}^2$
		$-2M(\vec{A}\cdot\vec{r})r\mathcal{R}^3$
		$+3MV_r^2\mathcal{R}^3$
		$+2Mv^2r\mathcal{R}^3$
		$-4M(\vec{V}\cdot\vec{v})r\mathcal{R}^3$
$+2MV_r^2\mathcal{R}^2$		$-4M(\vec{V}\cdot\vec{v})(\vec{r}\cdot\vec{v})\mathcal{R}^3$
$-4M(\vec{V}\cdot\vec{v})r\mathcal{R}^2$		$+2M(\vec{V}\cdot\vec{A})r\mathcal{R}^2$
		$-4M(\vec{A}\cdot\vec{v})r\mathcal{R}^2$
		$-2MV^2(\vec{r}\cdot\vec{v})\mathcal{R}^3$
		$+4M(\vec{A}\cdot\vec{r})(\vec{r}\cdot\vec{v})\mathcal{R}^3$

$$\vec{V}\cdot\frac{d\vec{V}}{dt} = r\mathcal{R}(\vec{V}\cdot\vec{A}) - (\vec{r}\cdot\vec{v})\mathcal{R}(\vec{V}\cdot\vec{A})$$

$$-\left(\vec{V}\cdot\frac{d\vec{V}}{dt}\right)\vec{V} = -(\vec{V}\cdot\vec{A})\,r\,\mathcal{R}\,\vec{V} + (\vec{V}\cdot\vec{A})(\vec{r}\cdot\vec{v})\mathcal{R}\,\vec{V}$$

(4) (5)

$$-\frac{1}{2}V^2\frac{d\vec{V}}{dt} = -\frac{1}{2}V^2 r\,\mathcal{R}\vec{A} + \frac{1}{2}V^2(\vec{r}\cdot\vec{v})\mathcal{R}\vec{A}$$

(4) (5)

$$\frac{d}{dt}\left\{-\frac{1}{2}V^2\vec{V}\right\} =$$

Grados en c^{-1}

4 $\quad -(\vec{V}\cdot\vec{A})\,r\,\mathcal{R}\;\vec{V} + \left[-\frac{1}{2}V^2 r\,\mathcal{R}\right]\vec{A}$

5 $\quad +(\vec{V}\cdot\vec{A})(\vec{r}\cdot\vec{v})\mathcal{R}\qquad\quad +\frac{1}{2}V^2(\vec{r}\cdot\vec{v})\mathcal{R}$

Grado en c^{-1}

$\vec{V}$

4 + $+2M(\vec{V}\cdot\vec{r})\mathcal{R}^3$
$-2(\vec{A}\cdot\vec{v})r\mathcal{R}$

5 $+2MV^2\mathcal{R}^2$
$-4MV^2r\mathcal{R}^3$
$-2M(\vec{V}\cdot\vec{v})\mathcal{R}^2$
$+2M(\vec{V}\cdot\vec{v})r\mathcal{R}^3$
$+2(\vec{A}\cdot\vec{v})(\vec{r}\cdot\vec{v})\mathcal{R}$

6 $+MV^2(\vec{V}\cdot\vec{r})\mathcal{R}^3$
$+2M(\vec{V}\cdot\vec{r})v^2\mathcal{R}^3$
$-8M(\vec{V}\cdot\vec{r})(\vec{V}\cdot\vec{v})\mathcal{R}^3$
$+2M(\vec{V}\cdot\vec{r})(\vec{A}\cdot\vec{r})\mathcal{R}^3$
$+4MV^2(\vec{r}\cdot\vec{v})\mathcal{R}^3$
$-4M(\vec{V}\cdot\vec{A})r\mathcal{R}^2$
$-4M(\vec{V}\cdot\vec{v})(\vec{r}\cdot\vec{v})\mathcal{R}^3$

$$\frac{d}{dt}\left\{-2(\vec{v}\cdot\vec{v})\vec{v}\right\} =$$

$$+\left[\begin{array}{l} -2(\vec{v}\cdot\vec{v})r\mathcal{R} \\ \\ +2(\vec{v}\cdot\vec{v})(\vec{r}\cdot\vec{v})\mathcal{R} \end{array}\right]\vec{A}$$

$\frac{d}{dt}\{-\vec{V}$

Grado en c^{-1}

4

$+[+MV^2\mathcal{R}^3]\vec{r}+[\quad]\vec{V}+[\quad]$

$+MV^2\mathcal{R}^2$

$-2MV^2r\mathcal{R}^3$

5

$+\frac{1}{2}MV^4\mathcal{R}^3$

$+MV^2v^2\mathcal{R}^3$

$-4MV^2(\vec{V}\cdot\vec{v})\mathcal{R}^3$

$+MV^2(\vec{A}\cdot\vec{r})\mathcal{R}^3$

$+2MV^2(\vec{r}\cdot\vec{v})\mathcal{R}^3$

$-2MV^2r$

$$\frac{d}{dt}\left\{-\vec{V}^2\vec{v}\right\} =$$

$\vec{V}$	+	$\vec{A}$	+	$\vec{v}$
$-MV^2\mathcal{R}^2$			$-MV^2\mathcal{R}^2$	
$2MV^2r\mathcal{R}^3$			$+MV^2r\mathcal{R}^3$	
			$+2(\vec{V}\cdot\vec{A})(\vec{r}\cdot\vec{v})\mathcal{R}$	
$2MV^2(\vec{r}\cdot\vec{v})\mathcal{R}^3$	$-2MV^2r\mathcal{R}^2$		$-2MV^2(\vec{r}\cdot\vec{v})\mathcal{R}^3$	

Grado en c^{-1}

$$+\frac{d}{dt}\left(v^2\vec{V}\right) =$$

4

$$\left[\begin{array}{l} -2M(\vec{r}\cdot\vec{v})\mathcal{R}^3 \\ -2M(\vec{v}\cdot\vec{v})\mathcal{R}^2 \\ +4M(\vec{v}\cdot\vec{v})r\mathcal{R}^3 \\ +2M\dot{v}^2\mathcal{R}^2 \\ -2Mrv^2\mathcal{R}^3 \end{array}\right]\vec{V} + \left[\begin{array}{l} +rv^2\mathcal{R} \\ -(\vec{r}\cdot\vec{v})v^2\mathcal{R} \end{array}\right]\vec{A}$$

5

$$\left[\begin{array}{l} -MV^2(\vec{r}\cdot\vec{v})\mathcal{R}^3 \\ +2M(\vec{r}\cdot\vec{v})v^2\mathcal{R}^3 \\ +4M(\vec{V}\cdot\vec{v})(\vec{r}\cdot\vec{v})\mathcal{R}^3 \\ +4M(\vec{A}\cdot\vec{v})r\mathcal{R}^2 \\ -2M(\vec{A}\cdot\vec{r})(\vec{r}\cdot\vec{v})\mathcal{R}^3 \end{array}\right]$$

$$\frac{d}{dt}\left[2(\vec{V}\cdot\vec{v})\vec{r}\right] =$$

$$-2M(\vec{V}\cdot\vec{v})\mathcal{R}^3\,\vec{r} + \left[-2M(\vec{V}\cdot\vec{r})\mathcal{R}^2 + 4M(\vec{V}\cdot\vec{v})r\mathcal{R}^3\right]\vec{V} + \left[\quad\right]\vec{A}$$

$$-MV^2(\vec{V}\cdot\vec{v})\mathcal{R}^3 - 2M(\vec{V}\cdot\vec{v})v\mathcal{R}^3 + 8M(\vec{V}\cdot\vec{v})^2\mathcal{R}^3 - 2M(\vec{V}\cdot\vec{v})(\vec{A}\cdot\vec{r})\mathcal{R}^3$$

$$-4M(\vec{V}\cdot\vec{v})(\vec{r}\cdot\vec{v})\mathcal{R}^3 \qquad 2M(\vec{V}\cdot\vec{v})r\mathcal{R}^3$$

Grado 74 en c^{-1}

$\vec{v}$

$$-2M(\vec{v}\cdot\vec{r})\mathcal{R}^3$$
$$\pm 2(\vec{A}\cdot\vec{v})r\mathcal{R} \qquad 4$$

$$-2Mv^2\mathcal{R}^2$$
$$+4Mv^2r\mathcal{R}^3$$
$$+2M(\vec{v}\cdot\vec{v})\mathcal{R}^2$$
$$-2M(\vec{v}\cdot\vec{v})r\mathcal{R}^3$$
$$+2M(\vec{v}\cdot\vec{r})\mathcal{R}^2$$
$$-2M(\vec{v}\cdot\vec{r})r\mathcal{R}^3$$
$$-2(\vec{A}\cdot\vec{v})(\vec{r}\cdot\vec{r})\mathcal{R} \qquad 5$$

$$-Mv^2(\vec{v}\cdot\vec{r})\mathcal{R}^3$$
$$-2M(\vec{v}\cdot\vec{r})v^2\mathcal{R}^3$$
$$+8M(\vec{v}\cdot\vec{r})(\vec{v}\cdot\vec{v})\mathcal{R}^3$$
$$-2M(\vec{v}\cdot\vec{r})(\vec{A}\cdot\vec{r})\mathcal{R}^3$$
$$-4Mv^2(\vec{r}\cdot\vec{v})\mathcal{R}^3$$
$$+4M(\vec{v}\cdot\vec{A})r\mathcal{R}^3$$
$$+4M(\vec{v}\cdot\vec{v})(\vec{r}\cdot\vec{v})\mathcal{R}^3$$
$$+4M(\vec{v}\cdot\vec{v})(\vec{r}\cdot\vec{v})\mathcal{R}^3 \qquad 6$$

Grado en c^{-1}

$$-6M\frac{d}{dt}(\mathcal{R}\vec{V}) =$$

$$\Big[+6M(\vec{r}\cdot\vec{v})\mathcal{R}^3 \Big]\vec{V} + \Big[-6Mr\mathcal{R}^2\Big]\vec{A}$$

$$-6M(\vec{V}\cdot\vec{r})\mathcal{R}^3 \quad \textcircled{4}$$

$$+6M(\vec{r}\cdot\vec{v})\mathcal{R}^2$$

$$-6M(\vec{V}\cdot\vec{v})\mathcal{R}^2$$

$$-6M(\vec{A}\cdot\vec{r})r\mathcal{R}^3 \quad \textcircled{5}$$

$$+6MV^2 r\mathcal{R}^3$$

$$-6MV^2(\vec{r}\cdot\vec{v})\mathcal{R}^3$$

$$+6M(\vec{A}\cdot\vec{r})(\vec{r}\cdot\vec{v})\mathcal{R}^2 \quad \textcircled{6}$$

1

Grado en c^{-1}

$\frac{d}{dt}\{ 4M\mathcal{R}\vec{v}$

④ $-4M^2\mathcal{R}^4\vec{r} + \quad \vec{V} +$

$-4M^2\mathcal{R}^2$

$+8M^2 r\mathcal{R}^3$

⑤

$-2M^2V^2\mathcal{R}^3$

$-4M^2v^2\mathcal{R}^3$

$+16M^2(\vec{V}\cdot\vec{v})\mathcal{R}^3$

$-8M^2(\vec{r}\cdot\vec{v})\mathcal{R}^3$

$8M^2 r\mathcal{R}^2$

⑥ $-4M^2(\vec{A}\cdot\vec{r})\mathcal{R}^3$

$$\frac{d}{dt}\left\{ 4M\mathcal{R}\vec{v} \right\}$$

$$\Big]\vec{V} + \Big[\quad\Big]\vec{A} + \Big[\quad\Big]\vec{v}$$

$\mathcal{R}^2$

$\mathcal{R}^3$

$\cdot\vec{v})\mathcal{R}^3$

$$8M^2 r\mathcal{R}^2$$

$$-4M(\vec{r}\cdot\vec{v})\mathcal{R}^3$$
$$+4M(\vec{V}\cdot\vec{r})\mathcal{R}^3$$
$$+4M^2\mathcal{R}^2$$
$$-4M^2 r\mathcal{R}^3$$
$$+4M(\vec{V}\cdot\vec{v})\mathcal{R}^2$$
$$+4M(\vec{A}\cdot\vec{r})r\mathcal{R}^3$$
$$-4MV^2 r\mathcal{R}^3$$
$$+8M^2(\vec{r}\cdot\vec{v})\mathcal{R}^3$$
$$+4MV^2(\vec{r}\cdot\vec{v})\mathcal{R}^3$$
$$-4M(\vec{A}\cdot\vec{r})(\vec{r}\cdot\vec{v})\mathcal{R}^3$$

$$\nabla\left(-\frac{3}{2}V^2\right) =$$

$$\boxed{\nabla\left(-\frac{3}{2}V^2\right) = +3(\vec{V}\cdot\vec{A})\mathcal{R}\vec{r}} \quad (3)$$

$$+\nabla(3\vec{V}\cdot\vec{v}) =$$

$$-3(\vec{A}\cdot\vec{v})\mathcal{R}\vec{r}$$

$$\boxed{\nabla(3\vec{V}\cdot\vec{v}) = -3(\vec{A}\cdot\vec{v})\mathcal{R}\vec{r}} \quad (3)$$

$$\nabla(2M\mathcal{R}) =$$

$$\nabla(-2M\mathcal{R}) = \left[+2M\mathcal{R}^3\vec{r} - 2M\vec{V}^2\mathcal{R}^3\vec{r} + 2M(\vec{A}\cdot\vec{r})\mathcal{R}^3\vec{r} - 2M\mathcal{R}^2\vec{V}\right]$$

(2) (4) (4) (3)

Grado en $\vec{r}$

2

3

4

$$\nabla(-2M\mathcal{R}) = \left[2M\mathcal{R}^3 - 2M\vec{V}^2\mathcal{R}^3 + 2M(\vec{A}\cdot\vec{r})\mathcal{R}^3\right]\vec{r} + \left[-2M\mathcal{R}^2\right]\vec{V}$$

$$\nabla\left(-\frac{7}{8}V^4\right) = +\frac{7}{2}V^2(\vec{V}\cdot\vec{A})\mathcal{R}\vec{r} \quad (5)$$

$$\nabla\left[+\frac{1}{2}V^2(\vec{V}\cdot\vec{v})\right] =$$

$$-\frac{1}{2}V^2(\vec{A}\cdot\vec{v})\mathcal{R}\vec{r} - (\vec{V}\cdot\vec{A})(\vec{V}\cdot\vec{v})\mathcal{R}\vec{r}$$

(4) (5)

$$\nabla\left[\frac{1}{2}V^2(\vec{V}\cdot\vec{v})\right] =$$

$$-\frac{1}{2}V^2(\vec{A}\cdot\vec{v})\mathcal{R}\vec{r}$$
$$-(\vec{V}\cdot\vec{A})(\vec{V}\cdot\vec{v})\mathcal{R}\vec{r}$$

Grado en c^{-1}

4

5

$$\nabla(\vec{V}\cdot\vec{v})^2 =$$

$$-2(\vec{V}\cdot\vec{v})(\vec{A}\cdot\vec{v})\mathcal{R}\vec{r}$$

$$\nabla(\vec{V}\cdot\vec{v})^2 = -2(\vec{V}\cdot\vec{v})(\vec{A}\cdot\vec{v})\mathcal{R}\vec{r}$$

(5)

$$\nabla\left[\frac{1}{2}V^2 v^2\right] =$$

$$-(\vec{V}\cdot\vec{A})v^2\mathcal{R}\vec{r}$$

$$\boxed{\nabla\left[\frac{1}{2}V^2 v^2\right] = -(\vec{V}\cdot\vec{A})v^2\mathcal{R}\vec{r}} \quad (5)$$

$$\nabla\left[-(\vec{V}\cdot\vec{v})v^2\right] =$$

$$+(\vec{A}\cdot\vec{v})v^2\mathcal{R}\vec{r}$$

$$\boxed{\nabla\left[-(\vec{V}\cdot\vec{v})v^2\right] = +(\vec{A}\cdot\vec{v})v^2\mathcal{R}\vec{r}} \quad (5)$$

$$\nabla\left[-2MV^2\mathcal{R}\right] =$$

$$+4M(\vec{V}\cdot\vec{A})\mathcal{R}^2\vec{r} + 2MV^2\mathcal{R}^3\vec{r} \quad (5)\ (4)$$

$$+2MV^4\mathcal{R}^3\vec{r} + 2MV^2(\vec{A}\cdot\vec{r})\mathcal{R}^3\vec{r} \quad (6)\ (6)$$

$$-2MV^2\mathcal{R}^2\vec{V} \quad (5)$$

Grado en c^{-1}

4

5

6

$$\nabla[-2MV^2\mathcal{R}] =$$

$$\begin{bmatrix} +2MV^2\mathcal{R}^3 \\ +4M(\vec{V}\cdot\vec{A})\mathcal{R}^2 \end{bmatrix}\vec{r} \quad \begin{bmatrix} \cdot \\ -2MV^2\mathcal{R}^2 \end{bmatrix}\vec{V}$$

$$\begin{bmatrix} -2MV^4\mathcal{R}^3 \cdot \\ +2MV^2(\vec{A}\cdot\vec{r})\mathcal{R}^3 \end{bmatrix}$$

$$\nabla[+6M(\vec{V}\cdot\vec{v})\mathcal{R}] =$$

$$-6M(\vec{A}\cdot\vec{v})^{(5)}\mathcal{R}^2\vec{r} - 6M(\vec{V}\cdot\vec{v})^{(4)}\mathcal{R}^3\vec{r}$$

$$+6MV^2(\vec{V}\cdot\vec{v})^{(6)}\mathcal{R}^3\vec{r} - 6M(\vec{V}\cdot\vec{v})(\vec{A}\cdot\vec{r})^{(6)}\mathcal{R}^3\vec{r}$$

$$+6M(\vec{V}\cdot\vec{v})^{(5)}\mathcal{R}^2\vec{V}$$

Grado en c^{-1}

$$\nabla\left[6M(\vec{V}\cdot\vec{v})\mathcal{R}\right] =$$

4 $\quad -6M(\vec{V}\cdot\vec{v})\mathcal{R}^3$

5 $\quad -6M(\vec{A}\cdot\vec{v})\mathcal{R}^2$

6 $\quad +6M(V^2)(\vec{V}\cdot\vec{v})\mathcal{R}^3$

$\quad -6M(\vec{V}\cdot\vec{v})(\vec{A}\cdot\vec{r})\mathcal{R}^3$

$$\vec{r} + \left[+6M(\vec{V}\cdot\vec{v})\mathcal{R}^2\right]\vec{V}$$

$$\nabla\left[-2Mv^2\mathcal{R}\right]$$

Grado en c^{-1}

4 $\quad +2Mv^2\mathcal{R}^3$

6 $\quad -2MV^2v^2\mathcal{R}^3$

$\quad +2M(\vec{A}\cdot\vec{r})v^2\mathcal{R}^3$

$$\vec{r} + \left[-2Mv^2\mathcal{R}^3\right]\vec{V}$$

5

Grado en c^{-1}

$$\nabla\left[-2Mv^2\mathcal{R}\right]$$

4 $\quad +2Mv^2\mathcal{R}^3$

5

6 $\quad -2MV^2v^2\mathcal{R}^3$

$\quad +2M(\vec{A}\cdot\vec{r})v^2\mathcal{R}^3$

$$\vec{r} + \left[-2Mv^2\mathcal{R}^3\right]\vec{V}$$

Grado en c^{-1}

4

5

6

$$\nabla[2M^2\mathcal{R}^2]$$

$$\left[+4M^2\mathcal{R}^4\right]\vec{r} + \left[-4M^2\mathcal{R}^3\right]\vec{V}$$

$$-4M^2V^2\mathcal{R}^4$$

$$+4M^2(\vec{A}\cdot\vec{r})\mathcal{R}^4$$

$\frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \vec{v}}\right) - \nabla \mathcal{L}$ hasta

Orden en $\frac{1}{c}$	$\vec{r}$	$\vec{v}$
2	$-2M\mathcal{R}^3$ $+2M\mathcal{R}^3$	
	$+3(\vec{V}\cdot\vec{A})\mathcal{R}$ $-3(\vec{A}\cdot\vec{v})\mathcal{R}$	$-2M\mathcal{R}^2$ $+4Mr\mathcal{R}^3$ $-2M\mathcal{R}^2$
3		
4	$-MV^2\mathcal{R}^3$ $-2MV^2\mathcal{R}^3$ $+8M(\vec{V}\cdot\vec{v})\mathcal{R}^3$ $-2M(\vec{A}\cdot\vec{r})\mathcal{R}^3$ $+MV^2\mathcal{R}^3$ $-2M(\vec{V}\cdot\vec{v})\mathcal{R}^3$ $-4M^2\mathcal{R}^4$ $-2MV^2\mathcal{R}^3$ $+2M(\vec{A}\cdot\vec{r})\mathcal{R}^3$ $-\frac{1}{2}V^2(\vec{A}\cdot\vec{v})\mathcal{R}$ $+2MV^2\mathcal{R}^3$ $-6M(\vec{V}\cdot\vec{v})\mathcal{R}^3$ $+2MV^2\mathcal{R}^3$ $+4M^2\mathcal{R}^4$	$-4M(\vec{r}\cdot\vec{v})\mathcal{R}^3$ $-(\vec{V}\cdot\vec{A})r\mathcal{R}$ $+2M(\vec{V}\cdot\vec{r})\mathcal{R}^3$ $-2(\vec{A}\cdot\vec{v})r\mathcal{R}$ $-2M(\vec{r}\cdot\vec{v})\mathcal{R}^3$ $+6M(\vec{r}\cdot\vec{v})\mathcal{R}^3$ $-6M(\vec{V}\cdot\vec{r})\mathcal{R}^3$ $-2MV^2$

hasta términos de 4º grado en c^{-1}

$\vec{A}$	$\vec{v}$
$-3r\mathcal{R}$	
$+3(\vec{r}\cdot\vec{v})\mathcal{R}$	$+2M\mathcal{R}^2$
	$-2Mr\mathcal{R}^3$
$+4Mr\mathcal{R}^2$	$+4M(\vec{r}\cdot\vec{v})\mathcal{R}^3$
$-\frac{1}{2}V^2r\mathcal{R}$	$-2(\vec{V}\cdot\vec{A})r\mathcal{R}$
$-2(\vec{V}\cdot\vec{v})r\mathcal{R}$	$-2M(\vec{V}\cdot\vec{r})\mathcal{R}^3$
$+rv^2\mathcal{R}$	$+2(\vec{A}\cdot\vec{v})r\mathcal{R}$
$-6Mr\mathcal{R}^2$	$-4M(\vec{r}\cdot\vec{v})\mathcal{R}^3$
	$+4M(\vec{V}\cdot\vec{r})\mathcal{R}^3$

Grado en c^{-1}	$\vec{F}$	$\frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \vec{V}}\right) - \nabla_{\vec{V}} \mathcal{L}$ hasta
2		
3	$+3(\vec{V}\cdot\vec{A})\mathcal{R}$ $-3(\vec{A}\cdot\vec{V})\mathcal{R}$	
4	$-\frac{1}{2}V^2(\vec{A}\cdot\vec{V})\mathcal{R}$	$-(\vec{V}\cdot\vec{A})r\mathcal{R}$ $-2(\vec{A}\cdot\vec{V})r\mathcal{R}$

hasta términos de 4. grado en c^{-1}

$\vec{A}$	$\vec{V}$
$-3r\mathcal{R}$	
$+3(\vec{r}\cdot\vec{v})\mathcal{R}$	
$-2Mr\mathcal{R}^2$	$-2(\vec{V}\cdot\vec{A})r\mathcal{R}$
$-\frac{1}{2}V^2 r\mathcal{R}$	$+2(\vec{A}\cdot\vec{v})r\mathcal{R}$
$-2(\vec{V}\cdot\vec{v})r\mathcal{R}$	
$+rv^2\mathcal{R}$	

$$\mathcal{R} = \left\{ r - (\vec{V}\cdot\vec{r}) \right\}^{-1} = r^{-1}\left\{ 1 - \frac{(\vec{V}\cdot\vec{r})}{r} \right\}^{-1}$$

$$\mathcal{R} = r^{-1}\left[1 + \frac{(\vec{V}\cdot\vec{r})}{r} + \frac{(\vec{V}\cdot\vec{r})^2}{r^2} + \frac{(\vec{V}\cdot\vec{r})^3}{r^3} + \frac{(\vec{V}\cdot\vec{r})^4}{r^4} + \cdots \right]$$

$$\mathcal{R} = \left[\frac{1}{r} + \frac{(\vec{V}\cdot\vec{r})}{r^2} + \frac{(\vec{V}\cdot\vec{r})^2}{r^3} + \frac{(\vec{V}\cdot\vec{r})^3}{r^4} + \frac{(\vec{V}\cdot\vec{r})^4}{r^5} + \cdots \right]$$

$$2M\mathcal{R} = \left[\frac{2M}{r} + \frac{2M(\vec{V}\cdot\vec{r})}{r^2} + \frac{2M(\vec{V}\cdot\vec{r})^2}{r^3} + \frac{2M(\vec{V}\cdot\vec{r})^3}{r^4} + \frac{2M(\vec{V}\cdot\vec{r})^4}{r^5} + \cdots \right]$$

Grado en c^{-1}

$\mathcal{L} =$

0	2	3	4
1	$+\frac{3}{2}V^2$	$+\frac{2M(\vec{V}\cdot\vec{r})}{r^2}$	
	$-3\vec{V}\cdot\vec{v}$		
	$+v^2$		
	$+2Mr$		

$\mathcal{L}$ en función de $r; V; v; (\vec{V}\cdot\vec{r}); (\vec{V}\cdot\vec{v})$

Grado en ε^{-1} →	0	1	2	3	4
→	1		$+\frac{3}{2}V^2$	$+\frac{2M(\vec{V}\cdot\vec{r})}{r^2}$	$+\frac{7}{8}V^4$
			$-3(\vec{V}\cdot\vec{v})$		$-\frac{1}{2}V^2(\vec{V}\cdot\vec{v})$
			$+v^2$		$-(\vec{V}\cdot\vec{v})^2$
			$+\frac{2M}{r}$		$-\frac{1}{2}V^2v^2$
					$+(\vec{V}\cdot\vec{v})v^2$
					$+\frac{2M(\vec{V}\cdot\vec{r})^2}{r^3}$
					$+\frac{2MV^2}{r}$
					$-\frac{6M(\vec{V}\cdot\vec{v})}{r}$
					$+\frac{2Mv^2}{r}$
					$+\frac{2M^2}{r^2}$

$$\mathcal{L} =$$

$$+ \frac{2M(\vec{V}\cdot\vec{r})^3}{r^5}$$

$$+ \frac{2M V^2(\vec{V}\cdot\vec{r})}{r^3}$$

$$- \frac{6M(\vec{V}\cdot\vec{r})(\vec{V}\cdot\vec{v})}{r^3}$$

$$+ \frac{2M(\vec{V}\cdot\vec{r})v^2}{r^3}$$

$$+ \frac{4M^2(\vec{V}\cdot\vec{r})}{r^4}$$

$$+ \frac{11}{6}V^6$$

$$- \frac{5}{8}V^4(\vec{V}\cdot\vec{v})$$

$$+ \frac{1}{2}V^2(\vec{V}\cdot\vec{v})^2$$

$$- (\vec{V}\cdot\vec{v})^3$$

$$- \frac{1}{8}V^4 v^2$$

$$- \frac{1}{2}V^2(\vec{V}\cdot\vec{v})v^2$$

$$+ (\vec{V}\cdot\vec{v})^2 v^2$$

$$+ \frac{2M(\vec{V}\cdot\vec{r})^4}{r^5}$$

$$+ \frac{2MV^2(\vec{V}\cdot\vec{r})^2}{r^4}$$

$$- \frac{6M(\vec{V}\cdot\vec{r})^2(\vec{V}\cdot\vec{v})}{r^4}$$

$$+ \frac{2M(\vec{V}\cdot\vec{r})^2 v^2}{r^4}$$

$$+ \frac{6M^2(\vec{V}\cdot\vec{r})^2}{r^5}$$

$$+ \frac{2MV^2(\vec{V}\cdot\vec{v})}{r^3}$$

$$- \frac{2M(\vec{V}\cdot\vec{v})^2}{r^3}$$

$$- \frac{6M(\vec{V}\cdot\vec{r})^2(\vec{V}\cdot\vec{v})}{r^3}$$

$$+ \frac{2M(\vec{V}\cdot\vec{r})^2 v^2}{r^3}$$

$$+ \frac{6M^2(\vec{V}\cdot\vec{r})^2}{r^4}$$

$$+ \frac{2MV^2(\vec{V}\cdot\vec{v})}{r}$$

$$- \frac{2M(\vec{V}\cdot\vec{v})^2}{r}$$

$$+ \frac{2M(\vec{V}\cdot\vec{v})v^2}{r}$$

$$- \frac{MV^2v^2}{r}$$

$$+ \frac{M^2V^2}{r^2}$$

$$- \frac{6M^2(\vec{V}\cdot\vec{v})}{r^2}$$

$$+ \frac{2M^2v^2}{r^2}$$

$$+ \frac{4}{3}\frac{M^3}{r^3}$$